

A 17 stage combined order 9 and 8 Runge-Kutta scheme

The scheme considered here may be regarded as a modification of the scheme suggested by Philip W. Sharp in:
See: Journal of Applied Mathematics & Decision Sciences, 4(2), 183-192 (2000),

"High order explicit Runge-Kutta pairs for ephemerides of the Solar System and the Moon".

The nodes with c_{11} as a parameter are as follows.

$$c_2 = \frac{19}{441}, c_3 = \frac{309752}{4945515}, c_4 = \frac{154876}{1648505}, c_5 = \frac{487}{2500}, c_6 = \frac{31}{145}, c_7 = \frac{10}{277}, c_8 = \frac{255330959}{412986350},$$

$$c_9 = \frac{26}{35}, c_{10} = \frac{16}{35}, c_{12} = \frac{31}{34}, c_{13} = \frac{560132958 c_{11}^2 - 221755475 c_{11} + 21116619}{2(1777345647 c_{11}^2 - 579547087 c_{11} + 47224747)}, c_{14} = \frac{73}{75}, c_{15} = \frac{97}{98}, c_{16} = 1, c_{17} = 1.$$

We can chose c_{11} so that that the order 10 quadrature condition is satisfied.

The principal error norm of the order 9 scheme, that is, the 2-norm of the principal error terms is: $0.3787157818 \times 10^{(-7)}$.

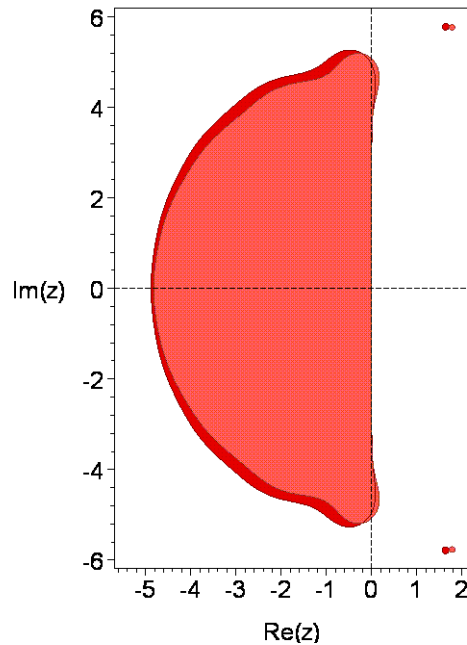
Note: 323 of the 719 principal error terms are zero.

The 2-norm of the order 8 embedded scheme is: $0.5435187750 \times 10^{(-6)}$.

The maximum magnitude of the linking coefficients is: 43.11506331.

The 2-norm of the linking coefficients is: 63.94184435.

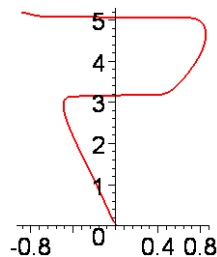
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 8 scheme appears in the darker shade.

The real stability intervals of the order 9 and 8 schemes are respectively $[-4.8221, 0]$ and $[-4.8781, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 9 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region of the order 9 scheme intersects the nonnegative imaginary axis in the interval $[3.1730, 5.0794]$ as well as at the origin.

a[10,5]=0.,
a[10,6]=.3931083273169945012929144960140970560902320536602677991740859986959068706131589825429,
a[10,7]=-0.1629580987593378496238252378774229890872985415499631895842836118430257745594345122869,
a[10,8]=.7100236638426404794052132767627667171764854591554011501479853558635508716820767638044e-1,
a[10,9]=-0.2365013517684546409932540395555709324896471557231157202605415744242405742517999680348e-1,
a[11,1]=.5660048451917660088120568857814639456092988537534102198290252970637380056604020723738e-1,
a[11,2]=0.,
a[11,3]=0.,
a[11,4]=0.,
a[11,5]=0.,
a[11,6]=.1008041833608547829672389338785642863537405465228148990025770177974082658428461585633,
a[11,7]=.1900733034348624395134636622343552549049131917018262196547366271045659316481247436964e-1,
a[11,8]=.5106444837921107348087916844602625268187673018241115496802873931583660668238847520617e-1,
a[11,9]=-0.1483815587765714404447209825957198212891808161746871699436869185447802671834395609524e-1,
a[11,10]=-0.7194250753852695410942591642876011194279308023719802776577936466449058247651985082868e-1,
a[12,1]=-0.5068797721570777629394861738567097771069249428690089360314550945864810649269633925843,
a[12,2]=0.,
a[12,3]=0.,
a[12,4]=0.,
a[12,5]=0.,
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a[12,7]=8.68873499673131205546614313049177467203622207563380553113666949240788457597884357890,
a[12,8]=-1.630215575001830447207304515931629018835122266688774015267131408819781322334781729527,
a[12,9]=.9234584288594656023822384563467220144985310298975900852612731287292749544349509191077,
a[12,10]=1.875192080089595397921845331992280560867832018231120037950385985393199395727501135526,
a[12,11]=-5.409740697839284657886128544940780104844046871221385109100202405153898582563703264689,
a[13,1]=1.525813559115217427647496766748962096805862079355954712638458197140904786323621814641,
a[13,2]=0.,
a[13,3]=0.,
a[13,4]=0.,
a[13,5]=0.,
a[13,6]=.8950781189875948479319392617564374381752850225128240002030890101479788450897682690882,
a[13,7]=-2.252478783341277162431559455767591359585575908679747150025191990615179765105211409235,
a[13,8]=.7199373219429047534986021860965886019849202768447683128506146863044835622074210383425,
a[13,9]=-2.197068022453065488614901970792017152090788733514803901247843160978826227470668825374,
a[13,10]=-0.6771403881843413787790662863348935191131590153331625893315401709400753668391739842712,
a[13,11]=.5840112043582632294569000580603652248745004089246300695984668301217699381179871386554,
a[13,12]=.3141693712476414499339539143513036352313308664085318588175618197394378937821044959613e-2,
a[14,1]=-1.726387910234743491825321576834052918547893378305369649513666998667023106004908903354,
a[14,2]=0.,
a[14,3]=0.,
a[14,4]=0.,
a[14,5]=0.,
a[14,6]=-3.877235295650650750111457736674914404811306880646602690947701108775298412978876469203,
a[14,7]=2.232687804942464639702150226893141189693007819859205673719851821276058071098049521529,
a[14,8]=-1.919287637840866102867223844146826231599511460726673770920978464628484143338535874168,
a[14,9]=1.515645681417394485468164502226083908945522593786392983722975796532640285919007335226,
a[14,10]=3.98430174633815594574936173122496664879953205207530433400690064502658145212239660336,
a[14,11]=2.226908107816131028717918317941921689559916589722074842538948978785809148582273001148,
a[14,12]=-0.1576356520097720330333171466150521744191639470256832777825134395894912239423704962431e-1,
a[14,13]=-1.447535598253575218196926572635481331264017607728430060888535411734077532761677888557,
a[15,1]=.5790007271453387121474328847859756860791682956945358669981133500927428514901215362065,
a[15,2]=0.,
a[15,3]=0.,
a[15,4]=0.,
a[15,5]=0.,
a[15,6]=-6.926400033664746870878678187466411619717377982733342394213474569611733358058726522041,
a[15,7]=-2.110998170201006557677125237026713062977891834839800909758892839794672421408895176523,

a[15,8]=-1.863516872140222529601728348125050054179531840083976399127257545390709568722470985953,
a[15,9]=1.676854971391547165014139975389710341812323942121081400997162480873432424261145102528,
a[15,10]=4.882660719390208321168190848944246226765546944336810453574115078922271717610837438435,
a[15,11]=6.906061409203239216025443633225982813456074287701093401086783237375232663556094424463,
a[15,12]=.3652130436003145709817543943421804359782079850077952553390874192328970207884517320943e-2,
a[15,13]=-2.144199784653917076610838865269793515184473022753084508048027979682316218705464819830,
a[15,14]=-1.1331917853909658652114404431973596735239637949747649671497331146637297859787245338203e-1,
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a[16,4]=0.,
a[16,5]=0.,
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a[16,8]=-1.772805688195569841262890127739004614683043329651879904748711942069314949208772368495,
a[16,9]=1.770791130274636273506752561876706814613230818333766847116210913697627624809622346057,
a[16,10]=5.516063190914566800659262227992321545758499694940548360754723281349419055518352841181,
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a[16,12]=.1112633608713130860435398091467365665174664908043760435288480602815071628259541202454e-1,
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a[16,14]=.5802885509454546816143268366808263773922741408290628111834398807255448094546524138911e-2,
a[16,15]=-2.126515165655050215699915849031910420322519375968774545498273724432937453053397151223e-1,
a[17,1]=.5208161323657053162806539967712301144070823976393961877512371346196743070120709768590,
a[17,2]=0.,
a[17,3]=0.,
a[17,4]=0.,
a[17,5]=0.,
a[17,6]=-9.271614096126431047611918435303243200586293817425463186204484662181591608331040597746,
a[17,7]=-2.427913400175018285599016020457878769122871981254083097948320679968217665515646392105,
a[17,8]=-2.559553922626221895445228649890365120433664063118761655889319835414504351668506198269,
a[17,9]=2.095837988927442665770321332633469410916626564735344329029161336717405585915874660479,
a[17,10]=6.481749926686284521625827232820268694497091893808605304365236212005098798017264692907,
a[17,11]=8.96460928827466326199642282660610735603285563744174221043877872884856712840844767056,
a[17,12]=.5082864121547591619301007092345472400339121541333417730643702767718480326726133156050e-1,
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a[17,14]=-5.924153721077765095733402065390029380239909436423310573417433979338505810922820189017e-1,
a[17,15]=0.,
a[17,16]=0.,

b[1]=.3961661035109776414988839445650902646379174873967661234230444774058857470349187194647e-1,
b[2]=0.,
b[3]=0.,
b[4]=0.,
b[5]=0.,
b[6]=0.,
b[7]=0.,
b[8]=0.,
b[9]=.6890847649364925763974247979716900210583634832794280707896110611961410509394937920555,
b[10]=.4994570598273576703670198120194537069582599255158708205912609899171118690291232599217,
b[11]=.2289101268353243344073825574401188230065263573412723277604860009214073362045331179334,
b[12]=-1.868439543337517513648899686499276426448246446940127935037062545725743691159298538190,
b[13]=-0.3397632033302333507370633358836703687382595475573608184431933601646380481229517358536,
b[14]=12.628490485990959929905393898183529942749003626155488530281242399279894497982938471118,
b[15]=-20.91195796330495476728149381013835453466302360901360042905069742028221250951429301476,
b[16]=10.03460166203147335644034737244999980961358446247935282076604842711745091993696277577,
b[17]=0.,

b*[1]=.3951259593918623677547024951806711518097212829507533069605536817237293157756048570795e-1,
b*[2]=0.,
b*[3]=0.,
b*[4]=0.,
b*[5]=0.,
b*[6]=0.,
b*[7]=0.,
b*[8]=0.,
b*[9]=.6403906092145461858717493760036246917636783159428388350950991879200328826453272704503,
b*[10]=.4920449125434706335473236316496820702667463435027234861003716023250217176315443546652,
b*[11]=.2293693857503113252881713722558455564932980790904691923380500383357827887439444002023,
b*[12]=-1.537071476431276312181727848474887029436530511337616705810141637305095432843555327816,
b*[13]=-3.178284396477701794144073982050761411925546843861262504568768950138939591600426461329,
b*[14]=10.1725370449233775152324317592463798202099318038602474346452674754873098041834777290,
b*[15]=-16.67187889873559369108734025641017769532855190759519243670646599955422963193863366559,
b*[16]=8.,
b*[17]=-4707573355625171403167088558345838795698956737241888590135914036730110083962264439137e-1.

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