

A modification of the Tsitouras-Papakostas 13 stage combined order 7 and 8 Runge-Kutta scheme

This scheme is a modification of the scheme of Tsitouras and Papakostas considered elsewhere.

It has similar characteristics to those of the Tsitouras-Papakostas scheme but all the nodes are in the interval [0, 1].

The nodes of the scheme are:

$$c_2 = \frac{2}{25}, c_3 = \frac{24514}{238491}, c_4 = \frac{12257}{79497}, c_5 = \frac{50}{129}, c_6 = \frac{34}{73}, c_7 = \frac{23}{148}, c_8 = \frac{196}{197}, c_9 = \frac{14118145156}{16038718811}, c_{10} = \frac{39}{43}, c_{11} = \frac{20}{21}, c_{12} = 1, c_{13} = 1.$$

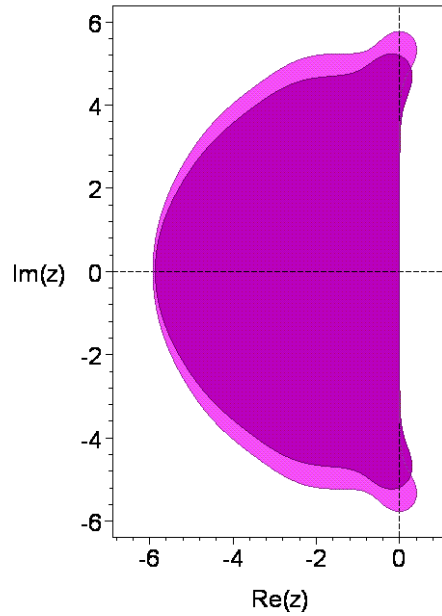
The principal error norm, that is, the 2-norm of the principal error terms is: $0.7313609930 \times 10^{(-6)}$.

The principal error norm of the order 7 embedded scheme is: $0.1012131360 \times 10^{(-4)}$.

The maximum magnitude of the linking coefficients is: 12.26567283.

The 2-norm of the linking coefficients is: 41.80047150.

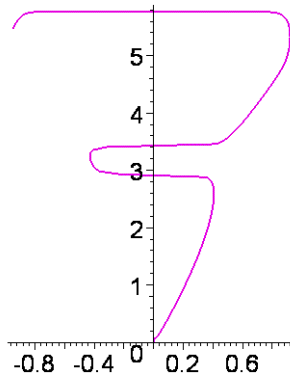
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 7 scheme appears in the darker shade.

The real stability intervals of the order 8 and 7 schemes are respectively [-5.9252, 0] and [-5.8669, 0].

The following picture shows the result of distorting the boundary curve of the stability region of the order 8 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the union of intervals: $[0, 2.9322] \cup [3.4087, 5.7689]$.

The coefficients in exact form are:

$c[2]=2/25,$
 $c[3]=24514/238491,$
 $c[4]=12257/79497,$
 $c[5]=50/129,$
 $c[6]=34/73,$
 $c[7]=23/148,$
 $c[8]=196/197,$
 $c[9]=14118145156/16038718811,$
 $c[10]=39/43,$
 $c[11]=20/21,$
 $c[12]=1,$
 $c[13]=1,$

$a[2,1]=2/25,$
 $a[3,1]=2090517149/56877957081,$
 $a[3,2]=3755851225/56877957081,$
 $a[4,1]=12257/317988,$
 $a[4,2]=0,$
 $a[4,3]=12257/105996,$
 $a[5,1]=14131686489425/35833975601529,$
 $a[5,2]=0,$
 $a[5,3]=-17700454220625/11944658533843,$
 $a[5,4]=17619604667500/11944658533843,$
 $a[6,1]=4418011/96055225,$
 $a[6,2]=0,$
 $a[6,3]=0,$
 $a[6,4]=9757757988/41995818067,$
 $a[6,5]=19921512441/106300094275,$
 $a[7,1]=480285555889619/7997789253816320,$
 $a[7,2]=0,$
 $a[7,3]=0,$
 $a[7,4]=52162106556696460538643/464887033284472899365888,$
 $a[7,5]=-16450769949384489/490009784398315520,$
 $a[7,6]=864813381633887/51718297665732608,$
 $a[8,1]=-41924855046865190259777/29067997485765047436800,$
 $a[8,2]=0,$
 $a[8,3]=0,$
 $a[8,4]=-1573638249639795538408923368457/337926761654282072170175876224,$
 $a[8,5]=-3144752379896908085076884133/286731750405790169489612800,$
 $a[8,6]=51820007951705750121680295/6052645677641770902708224,$
 $a[8,7]=19/2,$
 $a[9,1]=-14415222609369643343604689319116750509914398656839842108465299109803139261/$
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 $a[9,2]=0,$
 $a[9,3]=0,$
 $a[9,4]=-522416746733228002806292633565336738051126599108406039793785642181694795272896367/$
 $72447552632346584669609595477664264037757415889555156446859013382512921014002304,$
 $a[9,5]=-8249597526146667555031510474907199042876873605462323913470411629578679723249/$
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 $a[9,8]=3142544284003094691478527859401247056878071594901520381912/$
 $322898507178552018028498957174197954770661454622943631853501,$
 $a[10,1]=-9867878850858255039217746719915871073167304993320293/$
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a[10,3]=0,
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a[10,5]=-331923915046080335870716315310401046907/52747587227799808210222170585805619200,
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a[10,7]=2224286847389514767028684777161157168782986309/199937022215481792634238811967979191208436288,
a[10,8]=49853649680049123712136863581080955445/2729660625860504418029771633420026504704,
a[10,9]=-1203747916407015470833526610982521566345739350276392522818995/
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a[11,2]=0,
a[11,3]=0,
a[11,4]=-2302011554101675211827716537011959670051291/274508714084612705166461135767825539832704,
a[11,5]=-16889144669510407387099211531243899609/1891391571300777047888087779210764288,
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a[11,7]=956248096794157656802748368725840687390347399/77961324245284495254874844291292429525671778,
a[11,8]=10788313637659292115861359463369872242886/189380589527306698462795692309376152568545,
a[11,9]=89599398445595686856355698434884216540038527287573830949657578490242/
579385621338495813483588345965887373464382184677844152053023899833767,
a[11,10]=-1786335907413925929868932792832/7814458367137598885469908059185,
a[12,1]=-49410934155054900182882239365042643979034016237/31731623983903801998088697641133618410239344640,
a[12,2]=0,
a[12,3]=0,
a[12,4]=-1074611649128981113804182104383729704094059/244310054255337140402433369774092097581056,
a[12,5]=-222258155907862440804259443447899304774477/18911475471642929222051825301977449103360,
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a[12,10]=50908388575977059406815779042800/1299277084579288698410581464578279,
a[12,11]=-5373869958976816279363488081975/128249819495112776500552350596224,
a[13,1]=-68348459007984933111036701115865624594922076006017/
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a[13,3]=0,
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a[13,5]=-2984312771192069486473441797268836080579319/262376761616550996396340471077385680977920,
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a[13,7]=15921139337350831535408442010420239146408066675/1749382977105047216940523536513333290389079616,
a[13,8]=-64484096268292447966995856630949967425/5208159300615981523251328952205675621888,
a[13,9]=-842304890904016106357610472171364471743518622807702364576842637748625/
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a[13,10]=1124804893366359619795687870063500/18026098196604212076234696000197113,
a[13,11]=0,
a[13,12]=0,

b[1]=874545191389960761/19691638733588931200,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,

b[6]=420364804728976000977525051149/1187588452078929614685280764000,
b[7]=25102415824721153301532049778688/101007568454396515184681192806875,
b[8]=11291146602515386733913667333/8277895500290250100346304000,
b[9]=51522714764404063835592355866713645307758690796273878782033726047743318218807/
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b[10]=-582592205611226468655591/165538207004754039774500,
b[11]=43046993917969354593/35912185355757478400,
b[12]=-357120621384161/262158303907500,
b[13]=0,

b*[1]=13470793039761431/302948288209060480,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=225910982205515703574046329/636776649908273251841973600,
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b*[10]=-6457286835032584968329/2546741646226985227300,
b*[11]=0,
b*[12]=0,
b*[13]=-4954671690517567/1310791519537500.