

## A modification of Jim Verner's "Maple" 13 stage combined order 7 and 8 Runge-Kutta scheme

The nodes of the scheme are:

$$c_2 = \frac{3}{50}, c_3 = \frac{469}{4510}, c_4 = \frac{1407}{9020}, c_5 = \frac{100}{257}, c_6 = \frac{7}{15}, c_7 = \frac{12}{77}, c_8 = \frac{26}{27}, c_9 = \frac{7101051}{7944748}, c_{10} = \frac{11}{12}, c_{11} = \frac{19}{20}, c_{12} = 1, c_{13} = 1.$$

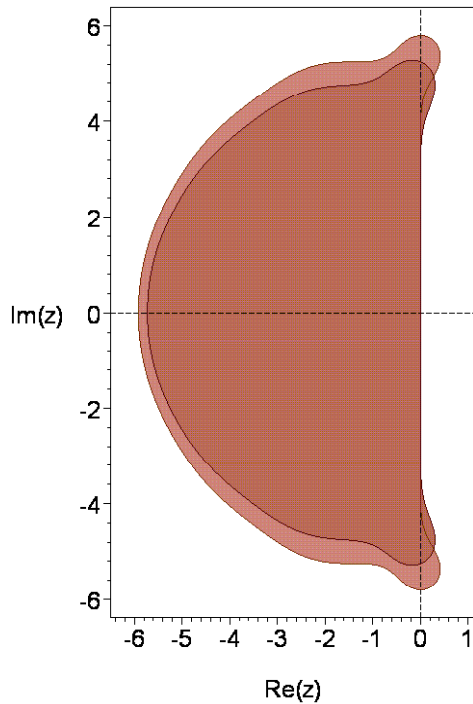
The principal error norm, that is, the 2-norm of the principal error terms is:  $0.6538793903 \times 10^{(-6)}$ .

The principal error norm of the order 7 embedded scheme is:  $0.9203287267 \times 10^{(-5)}$ .

The maximum magnitude of the linking coefficients is: 16.03848228.

The 2-norm of the linking coefficients is: 48.63317393.

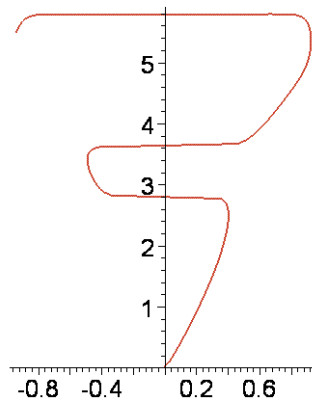
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 7 scheme appears in the darker shade.

The real stability intervals of the order 8 and 7 schemes are respectively  $[-5.9159, 0]$  and  $[-5.7261, 0]$ .

The following picture shows the result of distorting the boundary curve of the stability region of the order 8 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the union of intervals:  $[0, 2.80997] \cup [3.6379, 5.7964]$ .

The coefficients in exact form are:

$c[2]=3/50,$   
 $c[3]=469/4510,$   
 $c[4]=1407/9020,$   
 $c[5]=100/257,$   
 $c[6]=7/15,$   
 $c[7]=12/77,$   
 $c[8]=26/27,$   
 $c[9]=7101051/7944748,$   
 $c[10]=11/12,$   
 $c[11]=19/20,$   
 $c[12]=1,$   
 $c[13]=1,$

$a[2,1]=3/50,$   
 $a[3,1]=169309/12204060,$   
 $a[3,2]=219961/2440812,$   
 $a[4,1]=1407/36080,$   
 $a[4,2]=0,$   
 $a[4,3]=4221/36080,$   
 $a[5,1]=12985296430100/33603736057857,$   
 $a[5,2]=0,$   
 $a[5,3]=-16218027650000/11201245352619,$   
 $a[5,4]=48744170200000/33603736057857,$   
 $a[6,1]=3817807/81405000,$   
 $a[6,2]=0,$   
 $a[6,3]=0,$   
 $a[6,4]=10274191312/43991343405,$   
 $a[6,5]=40755997793/218862405000,$   
 $a[7,1]=15900802578/262289621825,$   
 $a[7,2]=0,$   
 $a[7,3]=0,$   
 $a[7,4]=6340315606886400/56274919101668113,$   
 $a[7,5]=-451131008276934/12909147305279225,$   
 $a[7,6]=984103149600/56231606512081,$   
 $a[8,1]=-6560178806172785891/5672858706535680000,$   
 $a[8,2]=0,$   
 $a[8,3]=0,$   
 $a[8,4]=-17/2,$   
 $a[8,5]=-5870605887517986369316613/652465735085103413760000,$   
 $a[8,6]=16204960050140425727/2255297505252764544,$   
 $a[8,7]=9658417761218465607893/776920972593666355200,$   
 $a[9,1]=-140461899806965182831986352616636648283813334213167183799/$   
 $231595702353396941018788535727415998878963658147758080000,$   
 $a[9,2]=0,$   
 $a[9,3]=0,$   
 $a[9,4]=-230388436455789139579016226863459716720858844533129533/$   
 $28162599104012402938735024801544932535316065892462592,$   
 $a[9,5]=-9790630998374957452129847257592953551765164772149796065413844391/$   
 $1879120863545568722091311485392002480259497346453409790689280000,$   
 $a[9,6]=197043235803985508254844363902235259599001596076904803130443/$   
 $44561965459500736173318259755870730512740108168829012803584,$   
 $a[9,7]=294620817758575088586344956781032015012776829359971133/$   
 $28166985596967449488723424839046646360350769597644800,$   
 $a[9,8]=126873616630173037746837292557884226969/13217298519653022949071050320940567220224,$   
 $a[10,1]=-41617847659972885613040591872450035471/52813005326633659912790449394810880000,$   
 $a[10,2]=0,$   
 $a[10,3]=0,$

a[10,4]=-269951087764496912154173876831/29845195411834947899362922496,  
a[10,5]=-2749986656729113102825386117572168116123/422416144552169230122612885222850560000,  
a[10,6]=324525706880938787969090744301341259977691/60253355178721829324099288547678254006272,  
a[10,7]=41487408138830374659637178881584696821/3493656315517022583105833038931558400,  
a[10,8]=932598433870374903885/57293130256189894463488,  
a[10,9]=-3345742205378079321732624680747620/191228884555191941725363563271035831,  
a[11,1]=-152032134715446654236835762683933540260220443/144031229933501187933392929852012953600000000,  
a[11,2]=0,  
a[11,3]=0,  
a[11,4]=-19926282430450459437356479596949/2204175587073348224489986560000,  
a[11,5]=-2861670807732347804933810531212726938261187/343166554822474341490702928140697600000000,  
a[11,6]=1334063326593468056350327316268714319601715109229/198658539882559017093840573946438829079429120000,  
a[11,7]=25587060085986906515751471801297011229903719273/2019042940187611700532950824131079569408000000,  
a[11,8]=1292083162729235236212706557/47701646396336675524288000000,  
a[11,9]=2091032384514247750201495665581688757138358851/60729869176412564938423797095435796334060237500,  
a[11,10]=-377694382258403/5816420236687500,  
a[12,1]=-240860055167915269684777564157507287321/117003858749851375574083424673098438400,  
a[12,2]=0,  
a[12,3]=0,  
a[12,4]=-98894702310834163626964955900/10545476035409921643978852639,  
a[12,5]=-252251601740617744737074278562623715189/16418177849613229685231088852333734400,  
a[12,6]=840813041585474673597744565360698902394175/70780871835683840841986501581774346003376,  
a[12,7]=24189514716339363218944405554841293171617/15082171924595104472807447927564153421312,  
a[12,8]=42847753308025275666/29882754550596969299,  
a[12,9]=-7717336222264885240564054011744574965898992045440/8479033418894797325547814554207128260520626293873,  
a[12,10]=16262726151696512/8940486723127047,  
a[12,11]=-16001553345352000000/6505707112968117383,  
a[13,1]=-2744022934559056024469622604306540529/1738222573548705964451797934060438400,  
a[13,2]=0,  
a[13,3]=0,  
a[13,4]=-35144243656562246120945761400/5953261868437390070644832157,  
a[13,5]=-54637123672393421308996942594910689159/4634295873089356958410234071688953600,  
a[13,6]=61239081290242266243670859206851281463175/6659681426411632770651421460598964953048,  
a[13,7]=5134209062671060909676995239537507099025/462964694026104436494336717002613458688,  
a[13,8]=-370363100142446513442/1535150001377392224067,  
a[13,9]=-212020682512329076068221666486441382648974080/390260289309289497513964664381911193640454941,  
a[13,10]=2242504532188281088/2908865886282319623,  
a[13,11]=0,  
a[13,12]=0,

b[1]=61903079921/1389136000824,  
b[2]=0,  
b[3]=0,  
b[4]=0,  
b[5]=0,  
b[6]=402653566621875/1136204715874336,  
b[7]=1872020731216849170881/7515476193013636507650,  
b[8]=-48961117784712267/15177413999438600,  
b[9]=1824678166477710343022780887139376956665357164944/332726480751222257863787417240798843822259974895,  
b[10]=-126172885488/13167945725,  
b[11]=41251039800000/5264998725539,  
b[12]=-107501739/674957600,  
b[13]=0,

b\*[1]=5709741119/127946736918,  
b\*[2]=0,  
b\*[3]=0,  
b\*[4]=0,

b\*[5]=0,  
b\*[6]=97458949528125/274256310728288,  
b\*[7]=79520000370367181779/319546003300661568600,  
b\*[8]=5689283182401789/2168201999919800,  
b\*[9]=1245827443533127167605703273831223125580576/298102207457268033088581737062702957958355,  
b\*[10]=-76671637728/13167945725,  
b\*[11]=0,  
b\*[12]=0,  
b\*[13]=-5522626587/8774448800.

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