Curtis 11 stage order 8 Runge-Kutta scheme with $c_2 = c_3,\ c_5 = \frac{5}{6} c_6$ and $c_6 < c_7$

See: An Eighth Order Runge-Kutta process with Eleven Function Evaluations per Step, by A. R. Curtis,
Numerische Mathematik, Vol. 16, No. 3 (1970), pages 268 to 277.

The nodes of the scheme are:

$$c_2 = \frac{134}{531} - \frac{16\sqrt{21}}{413},\ c_3 = \frac{134}{531} - \frac{16\sqrt{21}}{413},\ c_4 = \frac{67}{177} - \frac{24\sqrt{21}}{413},\ c_5 = \frac{5}{12} + \frac{5\sqrt{21}}{84},\ c_6 = \frac{1}{2} - \frac{\sqrt{21}}{14},\ c_7 = \frac{1}{2} + \frac{\sqrt{21}}{14},\ c_8 = \frac{1}{2},$$
$$c_9 = \frac{1}{2} - \frac{\sqrt{21}}{14},\ c_{10} = \frac{1}{2} + \frac{\sqrt{21}}{14},\ c_{11} = 1.$$

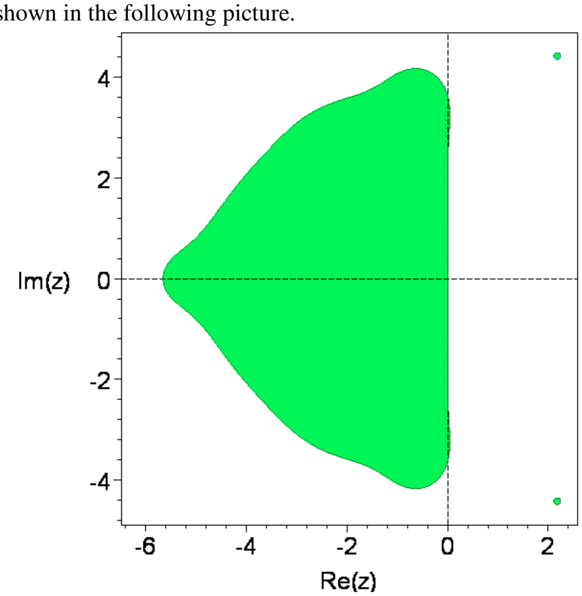
Note: The values:

$$\frac{1}{2},\quad \frac{1}{2} - \frac{\sqrt{21}}{14},\quad \frac{1}{2} + \frac{\sqrt{21}}{14}$$

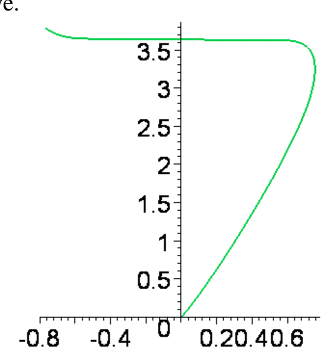
are the zeros of the derivative $P'_4(x) = \frac{d}{dx} P_4(x)$ of the **Legendre polynomial** $P_4(x)$ mapped linearly from the interval $[-1, 1]$ to the interval $[0, 1]$. They provide nodes for **Gauss-Lobatto integration** on the interval $[0, 1]$.

The principal error norm, that is, the 2-norm of the principal error terms is: $0.7786768212 \times 10^{(-4)}$.
The maximum magnitude of the linking coefficients is: 29.49644644.
The 2-norm of the linking coefficients is: 47.01200253.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-5.6583, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 8 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the union of intervals: $[0, 3.6398]$.

The coefficients in exact form are:
c[2]=134/531-16/413*21^(1/2),
c[3]=134/531-16/413*21^(1/2),
c[4]=67/177-24/413*21^(1/2),
c[5]=5/12+5/84*21^(1/2),
c[6]=1/2-1/14*21^(1/2),
c[7]=1/2+1/14*21^(1/2),
c[8]=1/2,
c[9]=1/2-1/14*21^(1/2),
c[10]=1/2+1/14*21^(1/2),
c[11]=1,

a[2,1]=134/531-16/413*21^(1/2),
a[3,1]=67/531-8/413*21^(1/2),
a[3,2]=67/531-8/413*21^(1/2),
a[4,1]=67/708-6/413*21^(1/2),
a[4,2]=0,
a[4,3]=67/236-18/413*21^(1/2),
a[5,1]=29562565/6946656+14672855/16208864*21^(1/2),
a[5,2]=0,
a[5,3]=-34079075/2315552-52274175/16208864*21^(1/2),
a[5,4]=6297425/578888+3615575/1519581*21^(1/2),
a[6,1]=-19/5380+361/37660*21^(1/2),
a[6,2]=0,
a[6,3]=0,
a[6,4]=434877/1541908-352987/10793356*21^(1/2),
a[6,5]=1587/7165-2423/50155*21^(1/2),
a[7,1]=242/1345+244/9415*21^(1/2),
a[7,2]=0,
a[7,3]=0,
a[7,4]=-39454899/63218228-59727347/442527596*21^(1/2),
a[7,5]=145059/637685+109883/4463795*21^(1/2),
a[7,6]=10461/14596+2275/14596*21^(1/2),
a[8,1]=10553/86080+1561/86080*21^(1/2),
a[8,2]=0,
a[8,3]=0,
a[8,4]=-43106763/126436456-6512171/63218228*21^(1/2),
a[8,5]=-590163/5101480+56221/1275370*21^(1/2),
a[8,6]=59073/116768+6943/58384*21^(1/2),
a[8,7]=21/64-5/64*21^(1/2),
a[9,1]=883/21520-1713/150640*21^(1/2),
a[9,2]=0,
a[9,3]=0,
a[9,4]=-39614935/758618736+170435971/1770110384*21^(1/2),
a[9,5]=1169591/3826110-685947/8927590*21^(1/2),
a[9,6]=-8113/1576368-240989/4729104*21^(1/2),
a[9,7]=-133/432+91/1296*21^(1/2),
a[9,8]=14/27-8/81*21^(1/2),
a[10,1]=-59929/69940-100901/489580*21^(1/2),
a[10,2]=0,
a[10,3]=0,
a[10,4]=930352614/205459241+1477486222/1438214687*21^(1/2),
a[10,5]=-5813547/8289905-834151/4463795*21^(1/2),
a[10,6]=-12380353/1423110-8028181/4269330*21^(1/2),
a[10,7]=35/156+35/468*21^(1/2),
a[10,8]=56/39+32/117*21^(1/2),
a[10,9]=297/65+63/65*21^(1/2),
a[11,1]=2621/5380+1561/16140*21^(1/2),

a[11,2]=0,
a[11,3]=0,
a[11,4]=-28737842/15804557-26048684/47413671*21^(1/2),
a[11,5]=-393442/637685+449768/1913055*21^(1/2),
a[11,6]=13486681/1970460+7145251/4597740*21^(1/2),
a[11,7]=31/54-5/42*21^(1/2),
a[11,8]=-16/27,
a[11,9]=-23/5-39/35*21^(1/2),
a[11,10]=13/18-13/126*21^(1/2),

b[1]=1/20,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=13/180,
b[7]=1/5,
b[8]=16/45,
b[9]=1/5,
b[10]=13/180,
b[11]=1/20.

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