Cooper-Verner 11 stage order 8 Runge-Kutta scheme with $c_7 < c_5$

See: Some Explicit Runge-Kutta Methods of High Order, by G. J. Cooper and J. H. Verner,
SIAM Journal on Numerical Analysis, Vol. 9, No. 3, (September 1972), pages 389 to 405

The nodes of the scheme are:

$$c_2 = \frac{1}{2},\ c_3 = \frac{1}{2},\ c_4 = \frac{1}{2} + \frac{\sqrt{21}}{14},\ c_5 = \frac{1}{2} + \frac{\sqrt{21}}{14},\ c_6 = \frac{1}{2},\ c_7 = \frac{1}{2} - \frac{\sqrt{21}}{14},\ c_8 = \frac{1}{2} - \frac{\sqrt{21}}{14},\ c_9 = \frac{1}{2},\ c_{10} = \frac{1}{2} + \frac{\sqrt{21}}{14},\ c_7 = 1.$$

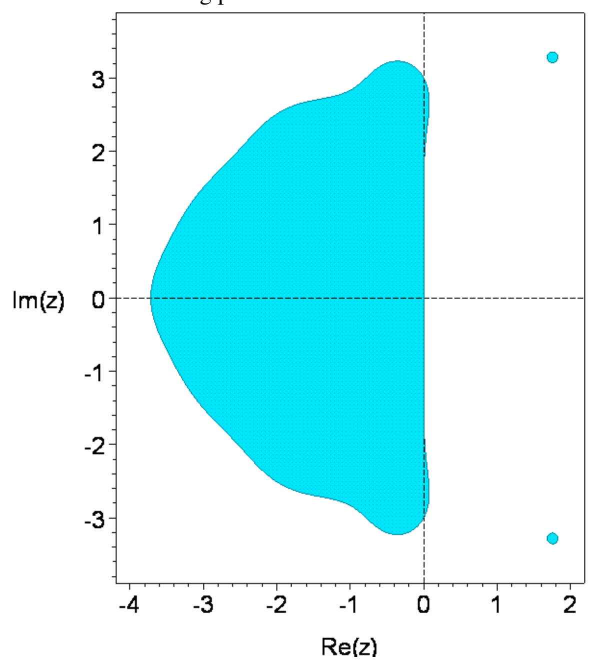
Note: The values:

$$\frac{1}{2},\quad \frac{1}{2} - \frac{\sqrt{21}}{14},\quad \frac{1}{2} + \frac{\sqrt{21}}{14}$$

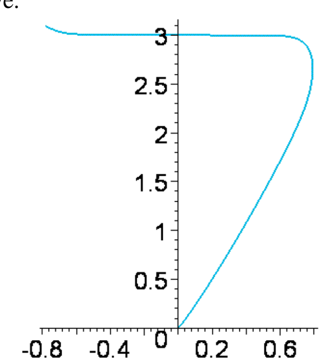
are the zeros of the derivative $P'_4(x) = \frac{d}{dx} P_4(x)$ of the **Legendre polynomial** $P_4(x)$ mapped linearly from the interval $[-1, 1]$ to the interval $[0, 1]$. They provide nodes for **Gauss-Lobatto integration** on the interval $[0, 1]$.

The principal error norm, that is, the 2-norm of the principal error terms is: 0.1226396827 x $10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 7.553840442.
The 2-norm of the linking coefficients is: 11.54549913.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-3.7154, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 8 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the union of intervals: $[0, 3.0018]$.

The coefficients in exact form are:
c[2]=1/2,
c[3]=1/2,
c[4]=1/2+1/14*21^(1/2),
c[5]=1/2+1/14*21^(1/2),
c[6]=1/2,
c[7]=1/2-1/14*21^(1/2),
c[8]=1/2-1/14*21^(1/2),
c[9]=1/2,
c[10]=1/2+1/14*21^(1/2),
c[11]=1,

a[2,1]=1/2,
a[3,1]=1/4,
a[3,2]=1/4,
a[4,1]=1/7,
a[4,2]=-1/14-3/98*21^(1/2),
a[4,3]=3/7+5/49*21^(1/2),
a[5,1]=11/84+1/84*21^(1/2),
a[5,2]=0,
a[5,3]=2/7+4/63*21^(1/2),
a[5,4]=1/12-1/252*21^(1/2),
a[6,1]=5/48+1/48*21^(1/2),
a[6,2]=0,
a[6,3]=1/4+1/36*21^(1/2),
a[6,4]=-77/120+7/180*21^(1/2),
a[6,5]=63/80-7/80*21^(1/2),
a[7,1]=5/21-1/42*21^(1/2),
a[7,2]=0,
a[7,3]=-48/35+92/315*21^(1/2),
a[7,4]=211/30-29/18*21^(1/2),
a[7,5]=-36/5+23/14*21^(1/2),
a[7,6]=9/5-13/35*21^(1/2),
a[8,1]=1/14,
a[8,2]=0,
a[8,3]=0,
a[8,4]=0,
a[8,5]=1/9-1/42*21^(1/2),
a[8,6]=13/63-1/21*21^(1/2),
a[8,7]=1/9,
a[9,1]=1/32,
a[9,2]=0,
a[9,3]=0,
a[9,4]=0,
a[9,5]=91/576-7/192*21^(1/2),
a[9,6]=11/72,
a[9,7]=-385/1152-25/384*21^(1/2),
a[9,8]=63/128+13/128*21^(1/2),
a[10,1]=1/14,
a[10,2]=0,
a[10,3]=0,
a[10,4]=0,
a[10,5]=1/9,
a[10,6]=-733/2205-1/15*21^(1/2),
a[10,7]=515/504+37/168*21^(1/2),
a[10,8]=-51/56-11/56*21^(1/2),
a[10,9]=132/245+4/35*21^(1/2),
a[11,1]=0,

a[11,2]=0,
a[11,3]=0,
a[11,4]=0,
a[11,5]=-7/3+7/18*21^(1/2),
a[11,6]=-2/5+28/45*21^(1/2),
a[11,7]=-91/24-53/72*21^(1/2),
a[11,8]=301/72+53/72*21^(1/2),
a[11,9]=28/45-28/45*21^(1/2),
a[11,10]=49/18-7/18*21^(1/2),

b[1]=1/20,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=0,
b[7]=0,
b[8]=49/180,
b[9]=16/45,
b[10]=49/180,
b[11]=1/20.

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