An 11 stage, order 7 Runge-Kutta scheme with an order 6 embedded scheme

See: Explicit Runge-Kutta Pairs with One More Derivative Evaluation than the Minimum, by P.W.Sharp and E.Smart, Siam Journal of Scientific Computing, Vol. 14, No. 2, pages. 338-348, March 1993.

The nodes of the scheme are:

$$c_2 = \frac{1}{200},\ c_3 = \frac{125788166632}{804556341815},\ c_4 = \frac{200814933981747403}{389065106257376584},\ c_5 = \frac{119}{198},\ c_6 = \frac{17}{19},\ c_7 = \frac{9526409}{47245149},\ c_8 = \frac{25}{29},\ c_9 = \frac{44}{45},\ c_{10} = 1,\ c_{11} = 1.$$

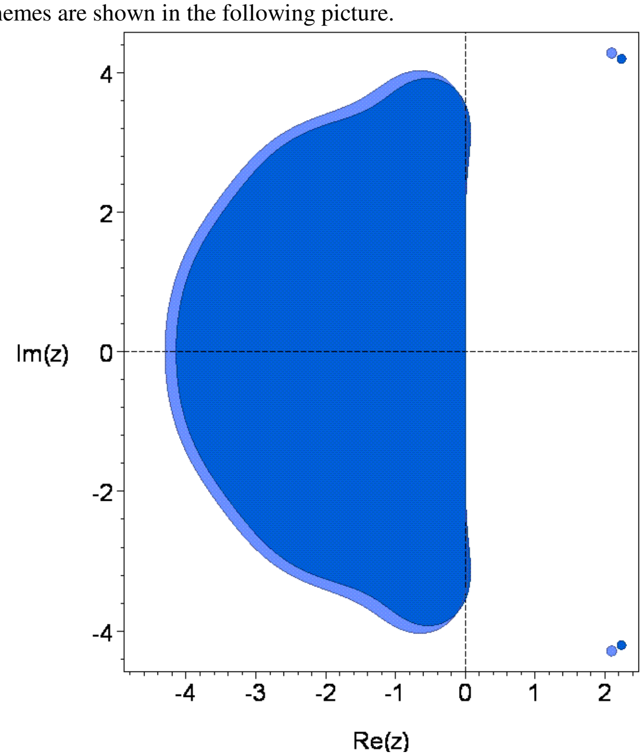
The principal error norm of the order 7 scheme, that is, the 2-norm of the principal error terms is: $0.2168941697 \times 10^{(-4)}$.
26 of the 115 principal error terms are zero.

An additional two error terms are (approximately) zero. They have a magnitude that is less than $0.2 \times 10^{(-27)}$.

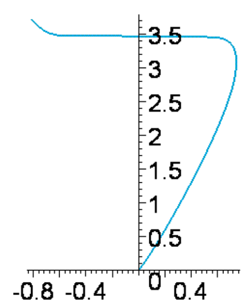
The 2-norm of the order 9 error terms is $0.8968841904 \times 10^{(-4)}$, which is approximately 4.135 times the principal error norm.

The principal error norm of the order 6 embedded scheme is: $0.3216449457 \times 10^{(-4)}$.
The maximum magnitude of the linking coefficients is: 10.33693692.
The 2-norm of the linking coefficients is: 24.18249843.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.
The real stability intervals of the order 7 and 6 schemes are respectively $[-4.3025, 0]$ and $[-4.1421, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 3.4593]$.

The coefficients in exact form are:
c[2]=1/200,
c[3]=125788166632/804556341815,
c[4]=200814933981747403/389065106257376584,
c[5]=119/198,
c[6]=17/19,
c[7]=9526409/47245149,
c[8]=25/29,
c[9]=44/45,
c[10]=1,
c[11]=1,

a[2,1]=1/200,
a[3,1]=-29621252385498445060506 4/129462181430947023498845,
a[3,2]=63290651458559192893696/25892436286189404699769,
a[4,1]=-12789372157071146745045111775696471200626449407/380815264047753329017214747410646491385300387 84,
a[4,2]=0,
a[4,3]=32445052113731709472131307285859833545001662335/38081526404775332901721474741064649138530038784,
a[5,1]=-431955631801634226455009883681953 3/58823738133898576978068703379693049 6,
a[5,2]=0,
a[5,3]=1223121779188860605856654584132649406757775688 75/32991318878010167844817832263254642776379666182 4,
a[5,4]=1564356728502081798022409036246134518927785330 0810312/
658363749533036108770050933585135314075673167927 26287,
a[6,1]=-3824213620692329526455584820908 99/72768912959378304431108675144702 88,
a[6,2]=0,
a[6,3]=2205734039259720200479905605749070291854455707011918 4193875/
4129997986457085733092356492524860864528295535072583956 7504,
a[6,4]=-94360812687526097028123187068745127462524044420500458282895614759616/
1170178792576501741359725978116830164978527431679907378255033642508 09,
a[6,5]=10429409903250357502298256364223097 60/85515506345393429514787820027554549 9,
a[7,1]=34593597270203433663140902653521768969908784170157140274555/
58731550602113257090244623300572326203937558128968689322244 8,
a[7,2]=0,
a[7,3]=517714047024312279768356989660598751616539775823173072267317364897221236919340917125/
333330780773975293533161697727890456259487743000308970714929289253541421782715374878 4,
a[7,4]=-304640485365550663069595193447523861946057797847478595200363312197854839340844066846348806 4/
511362639423029538585081763862319819637306053965614118086930675717683221504825261664507704 07,
a[7,5]=31957815155344547212722562970483115323606081283417524915395 2/
61539828207463833546666233046764838057663765046138457492520 65,
a[7,6]=-5919040014811009951092321586 6/11994398797423357814553515510595,
a[8,1]=-24687889053315680788516672392284637676310282602802 5/
94591686958853993706544353342277950256816041742718 4,
a[8,2]=0,
a[8,3]=4105216230872532336132293706540464473954640713998671441385245340625/
19499786542988241089964885103226653390881848969368448263921907226 72,
a[8,4]=-49324712599357751489276562537200199277679587257267614834494982647135327792 00/
10260713295176800963125818770711044624508389420487189702045571431972578553533,
a[8,5]=21269687576229892809323240545664650122099097000742563680/
20989526835414463140588698214708112363244319596705442863,
a[8,6]=-711581942949352658199 0/378493240381452882505729,
a[8,7]=-2838085862362767273670332075312660 0/1897075783840010072535356488629269 3,
a[9,1]=558353324326333774073044906290708552756484723746581752261/
584319440201571164550945793240631864149041989001022312500,
a[9,2]=0,
a[9,3]=-3614510090407314534068821036471754993634788628766244060214613687/
502464522228026334461988984585059909779594952312786529837471612,
a[9,4]=-13436380927846244044757803583952427765782251762574076119478709593044856969472/
66098728724720866795859088218193613387630021585614144986144311577612096231 25,

a[9,5]=989931864897123139010896933221647717910221123481001424643841888/
575874527302276156101678113881665298242482451696692517167915625,
a[9,6]=-372567259603706115805768838168/1683402581390679277326651009375,
a[9,7]=231443608884459407960125470613488480291827060789/31418904104758208921880920386941521649080865000,
a[9,8]=4101332345819049706502423/10667097363909719917125000,
a[10,1]=2693249580763133758720448819763251094287099002263195049/
1972459096581551425883657161923834583484247606165156240,
a[10,2]=0,
a[10,3]=-762720164986302748355429971639166755610094064685904534324565414570375/
73930254789526581156411334133121910635100300669232471904805722424944,
a[10,4]=-914106378133783649744437882531239843522925103950674595588223056094266864010432/
389018180564378499646810101592305688081747704847671122805431872050870556393341,
a[10,5]=3868116403444609936063780569978932749234663970855789282731320288/
2224720532488974689873485313867737218754855271707412737609361885,
a[10,6]=-2147337295919765809044534716/15282699757843572625171655145,
a[10,7]=76963822828349512771032624512097499610054141220500895/
7457737217451650694295122868750841403431006754989491,
a[10,8]=141940909070976808611971717/336796637787572286621096705,
a[10,9]=-545009894469844730599500000/14146180885244972240022216299,
a[11,1]=6936698309764946835725063360649553583306425655914739761/
5078529455579087652393405412856849782233648892058268000,
a[11,2]=0,
a[11,3]=-512606260594671504932297273531042063679229867086229919385168625/
49626125652150749082665578724450361459713081709904842453083616,
a[11,4]=-1548948850268913302819182056302536132743256445416938614472267815479730912/
652826950367613499218361365117961613704987867512238468998956163729502185,
a[11,5]=111870303021148790792075935935918275878271070980030897149566532/
63644799609999705548422499697144046294650653180108535971446675,
a[11,6]=-87122197804121104293267367/582943250254476957489125300,
a[11,7]=141145285619744149135929384507225418152792014862255637/
13654459409305766148231745889098379435265792575731520,
a[11,8]=60701635156866657458077/141408544478495930424000,
a[11,9]=-2472599248597125/64519956837334856,
a[11,10]=0,

b[1]=18530703372187/317986769215500,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=397406378366413672608/803680368165950784145,
b[6]=383280144717771439/119491812371674080,
b[7]=32739939833109197146986610130768938398484878542073/
102638217510541641713801705364029167418501950476800,
b[8]=-16070516558763250309/7373332958527296000,
b[9]=-68588512868559375/32259978418667428,
b[10]=1754022361907/1430294620800,
b[11]=0,

b*[1]=10644890436863/181706725266000,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=3667053975210116568/7266549440921797325,
b*[6]=839799471859174949/224047148196888900,
b*[7]=432325592089717918474482148525568808656853/
1360573252321546818820057421907905293476160,
b*[8]=-7030001847946787323/2764999859447736000,
b*[9]=-333800898560611875/129039913674669712,

b*[10]=0,
b*[11]=3/2.

Version: 27 Dec 2012, Peter Stone