

Tanaka-Yamashita Runge-Kutta scheme with an extended region of stability

See: On the Optimization of Some Nine-Stage Seventh-order Runge-Kutta Method, by M. Tanaka, S. Muramatsu and S. Yamashita, Information Processing Society of Japan, Vol. 33, No. 12 (1992) pages 1512-1526.

The nodes of the scheme are:

$$c_2 = \frac{1288}{14535}, c_3 = \frac{644}{4845}, c_4 = \frac{322}{1615}, c_5 = \frac{65}{258}, c_6 = \frac{627862490}{27724306937}, c_7 = \frac{78}{115}, c_8 = \frac{95}{124}, c_9 = 1, c_{10} = 1.$$

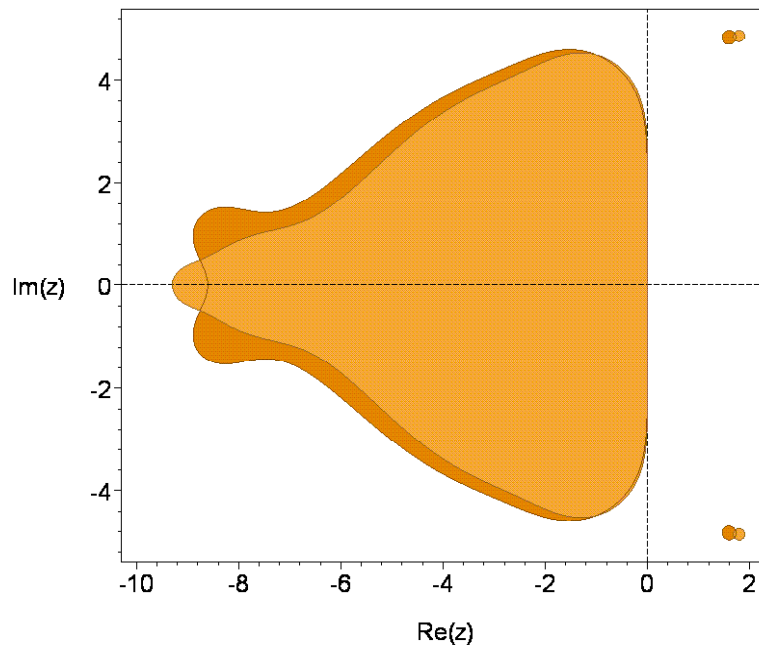
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1184005647 \times 10^{(-3)}$.

The principal error norm of the order 6 embedded scheme is: $0.1849301001 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 20.66712845.

The 2-norm of the linking coefficients is: 45.29041057.

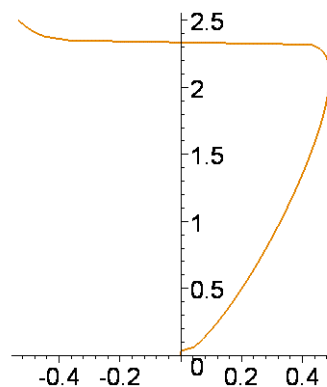
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-9.2990, 0]$ and $[-8.6059, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 2.3463]$.

The coefficients in exact form are:

$c[2]=1288/14535,$
 $c[3]=644/4845,$
 $c[4]=322/1615,$
 $c[5]=65/258,$
 $c[6]=627862490/27724306937,$
 $c[7]=78/115,$
 $c[8]=95/124,$
 $c[9]=1,$
 $c[10]=1,$

$a[2,1]=1288/14535,$
 $a[3,1]=161/4845,$
 $a[3,2]=161/1615,$
 $a[4,1]=161/3230,$
 $a[4,2]=0,$
 $a[4,3]=483/3230,$
 $a[5,1]=196347867755/3561236836416,$
 $a[5,2]=0,$
 $a[5,3]=134004261625/1187078945472,$
 $a[5,4]=149425089125/1780618418208,$
 $a[6,1]=874723327324627172137139789673935509613630495/
56881344496107103495850556251109088355454531158,$
 $a[6,2]=0,$
 $a[6,3]=140089490273660861720564275306545765967660125/
4375488038162084884296196634700699104265733166,$
 $a[6,4]=-2038049847879400647989164901369906650290192935250/
47909406273855748440601205051655304842157645301117,$
 $a[6,5]=2992403630086592541124850354372857004595944160/
168193972876080132251638040959092347542006316207,$
 $a[7,1]=-2001378790961964301303250341598299/131178829335937360185206084581250,$
 $a[7,2]=0,$
 $a[7,3]=-1366679891168526950613/3342867750190010177170,$
 $a[7,4]=-197077954039191584877658472075693196650/14482289235786224954374999272581521053,$
 $a[7,5]=2928205733652489758138852423071126752/289494157432907631631314224968221875,$
 $a[7,6]=32572843800597493853254181634376441943013874856495312/
1642002429836009758962688168840520197029337863346875,$
 $a[8,1]=226949925367094612475083609619198193642397605/120406368918742115270494114142742317844627456,$
 $a[8,2]=0,$
 $a[8,3]=83451940525721530822125/1129677771714575730562048,$
 $a[8,4]=7035716180093388934005544535766324331669337496890597125/
8744668672303692797949525789789288711635037496457428992,$
 $a[8,5]=1783910495800307104322539337559667105512922125/
384741540181237650862158213355907627413029681664,$
 $a[8,6]=-84153602056538973791098303633128803165153465256807063611390103929570560775/
37282901435832588263568764858094381410386083732352113498074163165795975168,$
 $a[8,7]=529655154424978769932790603243342890625/2074272966571578715335103162383459680256,$
 $a[9,1]=-2220302447236283385210081868020072818509/374126802552343922668161638021420098000,$
 $a[9,2]=0,$
 $a[9,3]=-580875348986851918117575/7422906155739208262352728,$
 $a[9,4]=-10152884092399228192381460837845336141124812794348025/
1298474693887469810743537803700532058113206856602944,$
 $a[9,5]=57628597675871150072147324302138021982593246488/
8694208064927983865022808524707850651868593125,$
 $a[9,6]=25006323928409346448859146781582297955220041834414805003311931721685248934905197/
3116797087097920659891402066923116953398124957660879445267880838924530153376000,$
 $a[9,7]=-991935992163983524020354479671037652370649875/
1096924827756227471690652450958041839154828032,$

a[9,8]=11028636941861502413824025771962099757599945728/
10166706010345110864067934052134974581343819375,
a[10,1]=2847557162233802909802913419338134005277/175580852316165047798631596921711256000,
a[10,2]=0,
a[10,3]=1552914837310075/7167358071597822,
a[10,4]=7813795507395804332400817811705117266280297151179075/
609385085239477995902119898391644620983028518989568,
a[10,5]=-306358654025510315315806741256227901425369583/37780230596405542839291492772111971777338125,
a[10,6]=-30230616135053261889365940573714713926595600173797519397657905897494488117634591/
1462739064177653822206630548583223553011887775071057363338006334613651993472000,
a[10,7]=-3556025825918703192187464108779875/11170666795578957880984290260063232,
a[10,8]=2140578935503723938488131712/2556174768949326564363043125,
a[10,9]=0,

b[1]=-28836965799708194669/40897924114041540000,
b[2]=0,
b[3]=0,
b[4]=-5319231056637407390089058139231875/4078513870347642725257280732562048,
b[5]=2430832495624902882205404599808/1640677233246140147577399278125,
b[6]=-58846832125102891510730576086257275195560457005949780449038492433452787/
55051022626529988904867724618756855605285697694381512902783594390080000,
b[7]=8765694250492187515737289375/142862058843931893355781359104,
b[8]=282726763309436004945812396032/864839130161188820942829590625,
b[9]=35795813026789129771/507885604115513709330,
b[10]=0,

b*[1]=-4303806316703372599/11685121175440440000,
b*[2]=0,
b*[3]=0,
b*[4]=-2188638181830974432849378205625/2703688346269567600435718085888,
b*[5]=1313681506776569792299836438/1214416900996402773928496875,
b*[6]=3205635250634133320066291736997892470430172563677735127041069/
5224293273794925935053617011014728265925425567271645535360000,
b*[7]=153260086062341088187716875/1103181921574763655256998912,
b*[8]=1144102534493369691260897984/4260291281582210940605071875,
b*[9]=0,
b*[10]=3/40.