

**Tanaka-Yamashita Runge-Kutta scheme with an order 6 embedded scheme ..  $c_8 \approx \frac{31}{34}$**

See: On the Optimization of Some Nine-Stage Seventh-order Runge-Kutta Method, by M. Tanaka, S. Muramatsu and S. Yamashita, Information Processing Society of Japan, Vol. 33, No. 12 (1992) pages 1512-1526.

The nodes of the scheme are:

$$c_2 = \frac{1607}{22500}, c_3 = \frac{1607}{15000}, c_4 = \frac{1607}{10000}, c_5 = \frac{891}{2000}, c_6 = \frac{4071252503200}{7099220853949}, c_7 = \frac{1729}{2000}, c_8 = \frac{9117}{10000}, c_9 = 1, c_{10} = 1.$$

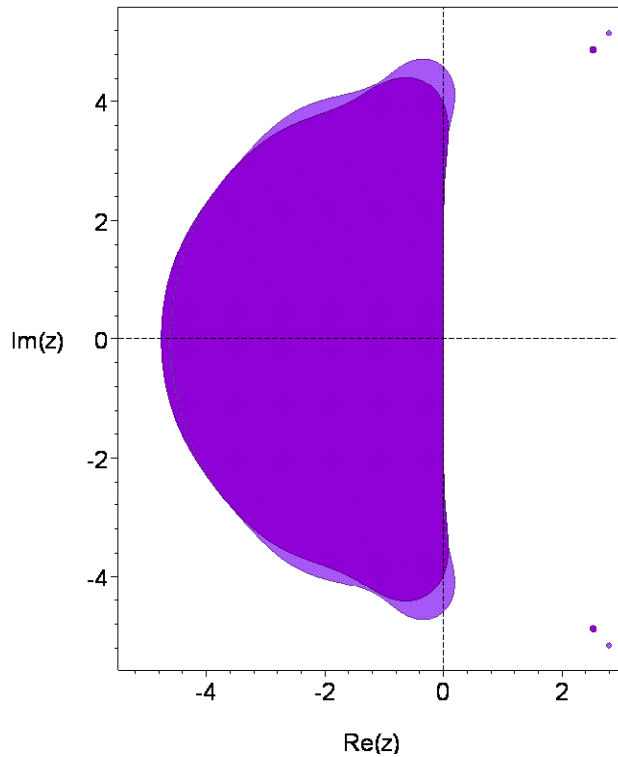
The principal error norm, that is, the 2-norm of the principal error terms is:  $0.1879311815 \times 10^{(-4)}$ .

The principal error norm of the order 6 embedded scheme is:  $0.1495154020 \times 10^{(-3)}$ .

The maximum magnitude of the linking coefficients is: 34.38994607.

The 2-norm of the linking coefficients is: 58.46932264.

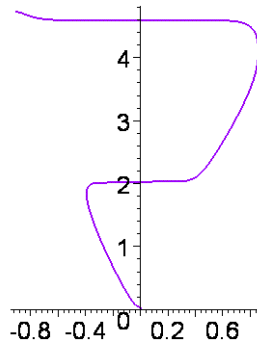
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively  $[-4.6143, 0]$  and  $[-4.7588, 0]$ .

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval:  $[2.0297, 4.5946]$ .

The coefficients in exact form are:

$c[2]=1607/22500,$   
 $c[3]=1607/15000,$   
 $c[4]=1607/10000,$   
 $c[5]=891/2000,$   
 $c[6]=4071252503200/7099220853949,$   
 $c[7]=1729/2000,$   
 $c[8]=9117/10000,$   
 $c[9]=1,$   
 $c[10]=1,$

$a[2,1]=1607/22500,$   
 $a[3,1]=1607/60000,$   
 $a[3,2]=1607/20000,$   
 $a[4,1]=1607/40000,$   
 $a[4,2]=0,$   
 $a[4,3]=4821/40000,$   
 $a[5,1]=12677077611/20659592000,$   
 $a[5,2]=0,$   
 $a[5,3]=-9738538227/4131918400,$   
 $a[5,4]=141310818/64561225,$   
 $a[6,1]=-9320865476627603487899920434921465901984671388929520969808800/$   
 $5844565175492066692734649271038266050018230856698229161364859,$   
 $a[6,2]=0,$   
 $a[6,3]=42855040209963551385363801323376855287780762639894400000000/$   
 $6559556874850804368950223648752262682399810164644477173249,$   
 $a[6,4]=-2888457498205410735105198070343993450034712794183864100000000/$   
 $583800561861721588836569904738951378733583104653358468419161,$   
 $a[6,5]=117375080543937799715109104139084137986385663360780000000/$   
 $201423648877013228781433354587992126253654010687584690099,$   
 $a[7,1]=98061073905500704312091365915742128519/30810786272884617043716423259200000000,$   
 $a[7,2]=0,$   
 $a[7,3]=-18730251559928245985427/1406557112674411969600,$   
 $a[7,4]=2620401649407750232469782193481341280681/229274335698377473839042511889140621300,$   
 $a[7,5]=-71807935028218160990510660176367/43601302843110574239319961363352,$   
 $a[7,6]=1114757789610634825138820774329056277348217560186161317194521/$   
 $916736419654616590150847449439932463149419009853787200000000,$   
 $a[8,1]=3281735816216626332653189642999932832673822471886127/$   
 $411797086290503570973077986451677902011560000000000,$   
 $a[8,2]=0,$   
 $a[8,3]=-241857116241174897550623/7032785563372059848000,$   
 $a[8,4]=56078728903289295546963821243325007671999118857246999/$   
 $1898149475891440333565434761911643516747286606847500,$   
 $a[8,5]=-5621750439254524942083709459826826069395610073/$   
 $1074506870877597588842990300143171797101285000,$   
 $a[8,6]=5883033372672881913690716274514884459726010317629788114471897179578037279801192876903891/$   
 $1844743001547346610690872025560093969400364679252549316211052148172533066742520000000000,$   
 $a[8,7]=-14931336612426939044612050793096808/88509786379714375814199416555224375,$   
 $a[9,1]=914643838795682022874504495621934002821662016714727/$   
 $193153475972934195127595915133824123962116702772800,$   
 $a[9,2]=0,$   
 $a[9,3]=-74922432690607762820843271000000/3511351700675223117910057271141,$   
 $a[9,4]=101991793380275119853483151250947735391497790461269671875/$   
 $5378226352346066387666756215278884585359641252408399826,$   
 $a[9,5]=-709426391608362228202469189878044393142541696875/$   
 $184085988657080970090110961585255999046123618242,$   
 $a[9,6]=12556457999345479663689547344977071497550676654983230186940227770077798262023126493154517031158756371/$   
 $5317304135058134494923216300847350933811706352046403016001543421317316805604238638163550577721105600,$

a[9,7]=5063124677045505250959296498321015317495590718750/  
13413671456497293886971818680717281240373128012913,  
a[9,8]=-10960256798784920020556964678332790323547660156250/  
44361585615230892202214356188214844991903399074547,  
a[10,1]=-3245092934737412156653883770745175456757315930727/  
12164107075702088223259428488035029681751631764800,  
a[10,2]=0,  
a[10,3]=7417325953962000000/9670080149636582291,  
a[10,4]=-41249039468855651588643051153721472138226105077846875/  
451601649225768631552820829162780108245260838184015288,  
a[10,5]=824065504357399468041349917265922117253283625/  
46372278125090303622179163785218836613140736488,  
a[10,6]=12516700857012319711199679182071393854410229018807524332988373569903512921757588314821197327844113/  
58468418048304631685960090335020099891322157250658516716439707478778710305041590880866459763371200,  
a[10,7]=900503945980037951479649424653238875/1654354119180517341218364134717009838,  
a[10,8]=-13152671671975380385320521875/71059534578099072728689192974,  
a[10,9]=0,

b[1]=3636236375323310491194413/78762914442172123767489600,  
b[2]=0,  
b[3]=0,  
b[4]=70968715478621650550908203125000/278889146803108156742141521429161,  
b[5]=15491691357263517950890625000/66889417091415666051423367611,  
b[6]=65368168566045079737703748391619787695701102658082358150696177058015599234607677873675041/  
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b[7]=30411876606181849400187500000/72183534289257775948592436369,  
b[8]=-1608888061926985613710937500000/8556213958972020166286257826733,  
b[9]=1359698371845958281425909/20232583896112271704624527,  
b[10]=0,

b\*[1]=62134654776957546682841011/1378351002738012165931068000,  
b\*[2]=0,  
b\*[3]=0,  
b\*[4]=3539232970023890145058984375/13682438640195660930291984567,  
b\*[5]=89332802800579086969884375/422205518160423500775440553,  
b\*[6]=20966717478772180960499156260306684425594123512751941894568243866068971675663/  
107542868875822105457671465974343777417120018181426377904108020353565225556000,  
b\*[7]=38071649981172839803512500/109677607993228658919068169,  
b\*[8]=-7447718953553443026795312500/67829555733640023968294229657,  
b\*[9]=0,  
b\*[10]=11/210.