An 11 stage order 7 Runge-Kutta scheme with a 12 stage FSAL order 6 embedded scheme .. $c_{10} = \frac{81}{82}$

The nodes of the scheme are:

$$c_2 = \frac{1}{1000},\ c_3 = \frac{28}{249},\ c_4 = \frac{14}{83},\ c_5 = \frac{415483}{780760},\ c_6 = \frac{87}{131},\ c_7 = \frac{109}{659},\ c_8 = \frac{7}{16},\ c_9 = \frac{13}{16},\ c_{10} = \frac{169}{170},\ c_{11} = 1,\ c_{12} = 1.$$

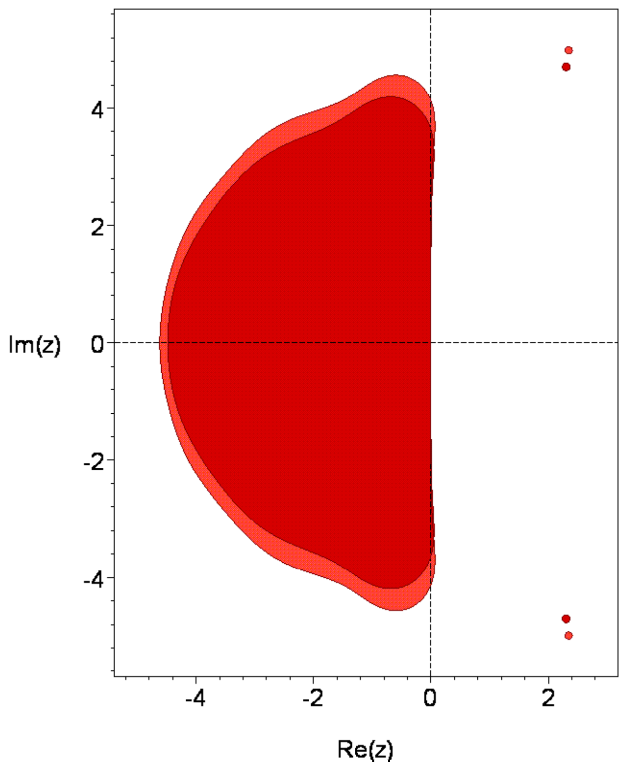
The principal error norm, that is, the 2-norm of the principal error terms is: $0.9367419806 \times 10^{(-5)}$.

The principal error norm of the order 6 embedded scheme is: $0.1815404680 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 22.37881168.

The 2-norm of the linking coefficients is: 47.66433339.

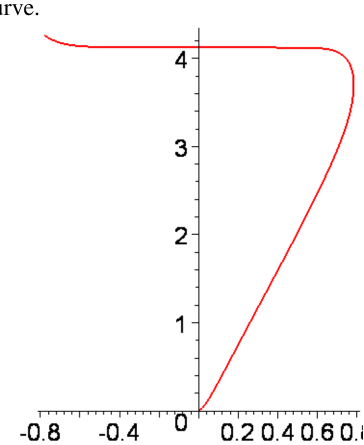
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6181, 0]$ and $[-4.4818, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 4.1292]$.

The coefficients in exact form are:
c[2]=1/1000,
c[3]=28/249,
c[4]=14/83,
c[5]=415483/780760,
c[6]=87/131,
c[7]=109/659,
c[8]=7/16,
c[9]=13/16,
c[10]=169/170,
c[11]=1,
c[12]=1,

a[2,1]=1/1000,
a[3,1]=-385028/62001,
a[3,2]=392000/62001,
a[4,1]=7/166,
a[4,2]=0,
a[4,3]=21/166,
a[5,1]=20184949896867695O243/18656867757700659200O,
a[5,2]=0,
a[5,3]=-777541397023211694969/186568677577006592000,
a[5,4]=337487397017512409163/93284338788503296000,
a[6,1]=206935503/6884238116,
a[6,2]=0,
a[6,3]=0,
a[6,4]=404414081147/1257566797788,
a[6,5]=1534888336000/4911984599757,
a[7,1]=39743335520386666547/572655525899435427204,
a[7,2]=0,
a[7,3]=0,
a[7,4]=212898348102841647147719/2010207409308970807088412,
a[7,5]=-49544686747746827756272000/2729832755347968158104608573,
a[7,6]=1077348693331815806/130730240620467840411,
a[8,1]=17125839160165511097181/583729762655220563902464,
a[8,2]=0,
a[8,3]=0,
a[8,4]=174040744000019283021419611/2049081587190964014926856192,
a[8,5]=128489749336355703667293557125/956526769655402159824357169664,
a[8,6]=-125509550840850981113069/2931683770285010740150272,
a[8,7]=19/82,
a[9,1]=304584927066618919841641919928442600454537797/1336435146974329082026479192445476428829818880,
a[9,2]=0,
a[9,3]=0,
a[9,4]=-183130124969528341358670952614060593600746254959/43039657262695209676619384887672437213535272960,
a[9,5]=-5992985997487815939072736862205038623064898960325/2245491222915232082243948587229615621971585089024,
a[9,6]=1322708054627964884973707202944501083084973588321/1346859151282526737213165214426258933563303526400,
a[9,7]=873822912254162442527647371013373/228741131636931363287619371663360,
a[9,8]=51299867691487048524852/18956439276208492414825,
a[10,1]=-1256795279708492037734149713024454589348072577632030104333/
1844322652469644188488724709227695887423797701298885000000,
a[10,2]=0,
a[10,3]=0,
a[10,4]=1155321277999101211105955197933773620274343648077208821293051/
59396084444375336903774156155177493077283524727141295000000,
a[10,5]=100171919666627080787492626771580340159478395878727610639533656/
8428868485387823508039378612742757916792962274222867420200625,

a[10,6]=-75691885617633891987993521638144638721175631419293321698496028553/
19653979525281151679347336094614453722396051644360672223450000000,
a[10,7]=-38688518491828194541031368152041107061501224313931/
23691978700786561492029010557625479030322285000000,
a[10,8]=-145680407248255277894206069634089984/14192495726244728880776189465890625,
a[10,9]=659660820257407968563 2/8396199709857954528125,
a[11,1]=-191391765077327664800004270192103183018029796185 6463/
242053599987401411448555153578493858708705077306713 2,
a[11,2]=0,
a[11,3]=0,
a[11,4]=794034598781907555787098179682463625970047118793613/
354815353965062564581692467789508565947227438684 52,
a[11,5]=1253876511431977346537823112898265251996802734164616 0000/
915484882132450383552157953702371732055721397872191177,
a[11,6]=-87976582885029217249010634051856922311708771953455789 1994/
19609686876602973131292668864051832099150560307928 1962497,
a[11,7]=-26773940308698113601982120741972828286815906/14220463788522665086401376326525424823847 89,
a[11,8]=-918990495679234610154177376026624/77529355682339127064062513128033,
a[11,9]=819316668609331200/920300389324474639,
a[11,10]=-689437436854468500 0000/9706884890396295379540 49,
a[12,1]=3042259121/61252403940,
a[12,2]=0,
a[12,3]=0,
a[12,4]=0,
a[12,5]=0,
a[12,6]=135669619784256976 07/751479213238067160 00,
a[12,7]=49289738735215769940 3367/197014283456489650582 2000,
a[12,8]=5410824060928/20472512295375,
a[12,9]=986554630144/6438843573435,
a[12,10]=269064197119512500/452758788468179739,
a[12,11]=-135068333/274428000,

b[1]=3042259121/61252403940,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=13566961978425697607/75147921323806716000,
b[7]=492897387352157699403367/1970142834564896505822000,
b[8]=5410824060928/20472512295375,
b[9]=986554630144/6438843573435,
b[10]=269064197119512500/452758788468179739,
b[11]=-135068333/274428000,

b*[1]=7832789069474402459 17/155359385168342027400 00,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=299178814602945973984800943303 3/19060370046834035632749036000000,
b*[7]=12357002663874347260355898575771781 7/49970312964637074580650705826200 0000,
b*[8]=17751175630255412885350 4/649075937937135306609375,
b*[9]=3724138845319784904550 4/2041419426868276965168 75,
b*[10]=19029906773175457653236565 5/459347372389706921889026076,
b*[11]=-229601674282232280 83/696053748269880000 00,
b*[12]=1/200.

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