An 11 stage order 7 Runge-Kutta scheme with a 12 stage FSAL order 6 embedded scheme .. $c_{10} = \frac{19}{20}$

The nodes of the scheme are:

$$c_2 = \frac{1}{20},\ c_3 = \frac{8}{57},\ c_4 = \frac{4}{19},\ c_5 = \frac{44881}{82208},\ c_6 = \frac{37}{56},\ c_7 = \frac{61}{367},\ c_8 = \frac{25}{57},\ c_9 = \frac{39}{50},\ c_{10} = \frac{19}{20},\ c_{11} = 1,\ c_{12} = 1.$$

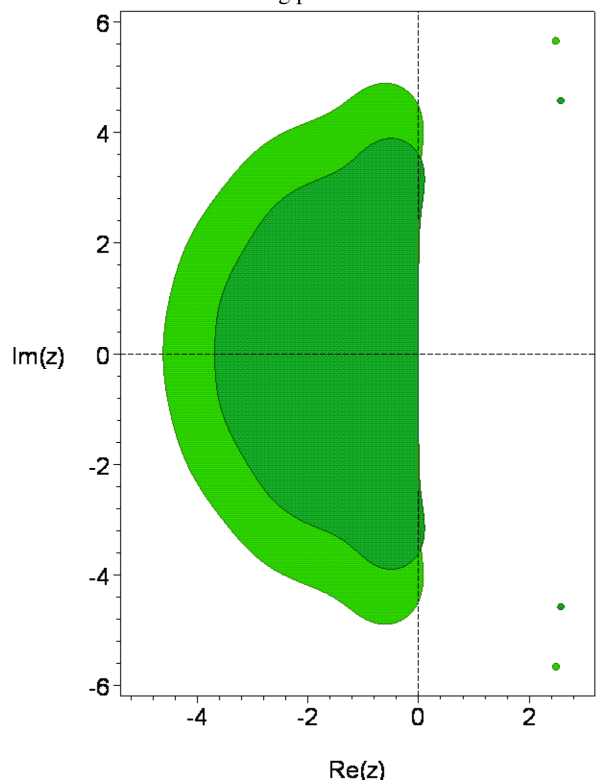
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1615220060 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.2712418538 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 10.12679588.

The 2-norm of the linking coefficients is: 17.79287306.

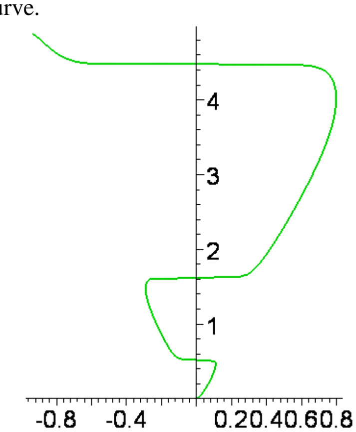
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6193, 0]$ and $[-3.6865, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the union of intervals: $[0, 0.5284] \cup [1.6031, 4.4761]$.

The coefficients in exact form are:
c[2]=1/20,
c[3]=8/57,
c[4]=4/19,
c[5]=44881/82208,
c[6]=37/56,
c[7]=61/367,
c[8]=25/57,
c[9]=39/50,
c[10]=19/20,
c[11]=1,
c[12]=1,

a[2,1]=1/20,
a[3,1]=-184/3249,
a[3,2]=640/3249,
a[4,1]=1/19,
a[4,2]=0,
a[4,3]=3/19,
a[5,1]=10879382999084569/17778381694173184,
a[5,2]=0,
a[5,3]=-41275080377096907/17778381694173184,
a[5,4]=20050852951463513/8889190847086592,
a[6,1]=5533979/91295232,
a[6,2]=0,
a[6,3]=0,
a[6,4]=12854870299/39431336448,
a[6,5]=3657883862/13345483011,
a[7,1]=158475499801357/1970033827325064,
a[7,2]=0,
a[7,3]=0,
a[7,4]=31153019468948131/297713925429454536,
a[7,5]=-3016545448677376/81418799560147215,
a[7,6]=150256844634176/8265656149993815,
a[8,1]=2924917590401981135/66605023970341440552,
a[8,2]=0,
a[8,3]=0,
a[8,4]=445876617351108665/3098009485599691752,
a[8,5]=196990339933642293248/1651616643670523695197,
a[8,6]=-7149897627279951488/167672519638445079477,
a[8,7]=35/201,
a[9,1]=-42211185912602462564070166033950198162012423/342205824198861190074733210927141821875000000,
a[9,2]=0,
a[9,3]=0,
a[9,4]=-2318859757393123763522345863545224823245606023/847779727566936317150916383049577334375000000,
a[9,5]=-31112530877798875425923566022361029283298688/28981348290292110337421437961130838134765625,
a[9,6]=15292107243270980188039355727714753150306881614/29901512151935859805913259351031993989013671875,
a[9,7]=40769840728265525109900718286661374039/15549375265121654840821192853125000000,
a[9,8]=945612121233663016296797907/598971783761117752865000000,
a[10,1]=53421576995301575930555089451778526350776610941/205195002329815636484038978294464618799958400000,
a[10,2]=0,
a[10,3]=0,
a[10,4]=502896807923589267114219976720637605496180343553/195519042445520666638376728307225007182256000000,
a[10,5]=363012698911770750340098104804203047936446648/417738629670365256911027966408581558444453125,
a[10,6]=-115813448945099442948683847145831524761051670902703/
191939521001616690365566010466177813315410464375000,
a[10,7]=-28699281021898330563484694969577298696532859/13463308844970886630335729997542445904000000,
a[10,8]=-16493845602934685528675361413563797/28001674400224052367433014064000000,

a[10,9]=1285791152889375/2245412845835068,
a[11,1]=89462997577799701289200380185298019046438429/84108584887856406378631238816963629525265400,
a[11,2]=0,
a[11,3]=0,
a[11,4]=6492689767645528199626511179748230475508923/641139590808912277141769373352511460044088,
a[11,5]=694071986047276723594566813393158946881536/175338845029991694153905751089388426975345,
a[11,6]=-187824233463716665067125996081330354271085997696/30211283627207706655231802682148684012056060045,
a[11,7]=-197008013321007326029676937491820383240505/20203428941091626061502271171655020032789,
a[11,8]=-17297582435764003600121633126256/9677847661048496307295108424975,
a[11,9]=165638979000000000/39827589068734103,
a[11,10]=-845624320000/1531259401089,
a[12,1]=125174203/2508655500,
a[12,2]=0,
a[12,3]=0,
a[12,4]=0,
a[12,5]=0,
a[12,6]=56446289600512/342606012403635,
a[12,7]=62143252047047649977/247198834682622733680,
a[12,8]=9636489994384107/36666550775008000,
a[12,9]=5373710937500/40041548453907,
a[12,10]=13212880000/87750675837,
a[12,11]=-98471/7223040,

b[1]=125174203/2508655500,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=56446289600512/342606012403635,
b[7]=62143252047047649977/247198834682622733680,
b[8]=9636489994384107/36666550775008000,
b[9]=5373710937500/40041548453907,
b[10]=13212880000/87750675837,
b[11]=-98471/7223040,

b*[1]=185777316884981989/3681484912241059950,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=22385886671596155610112/158772260638005081292791,
b*[7]=537978856569572663706487529/2159330585189973467574291759,
b*[8]=32498562725533982078667/120108582798663659157650,
b*[9]=1235450293578257031250/7631363446505532525519,
b*[10]=498401087642132308000/3863258097409608192999,
b*[11]=0,
b*[12]=-274194775/132076182681.

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