A Sharp-Verner 11 stage order 7 Runge-Kutta scheme with a 12 stage FSAL order 6 embedded scheme

See: Completely Imbedded Runge-Kutta Pairs, by P.W.Sharp and J.H.Verner, Siam Journal on Numerical Analysis, Vol.31, No.4. (August 1994) pages 1169-1190.

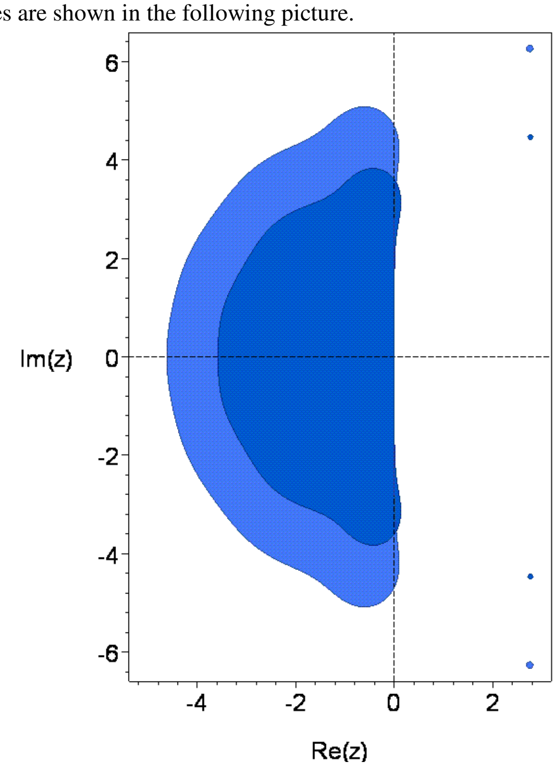
The nodes of the scheme are:

$$c_2 = \frac{1}{12},\ c_3 = \frac{4}{27},\ c_4 = \frac{2}{9},\ c_5 = \frac{5}{9},\ c_6 = \frac{2}{3},\ c_7 = \frac{1}{6},\ c_8 = \frac{4}{9},\ c_9 = \frac{3}{4},\ c_{10} = \frac{11}{12},\ c_{11} = 1,\ c_{12} = 1.$$

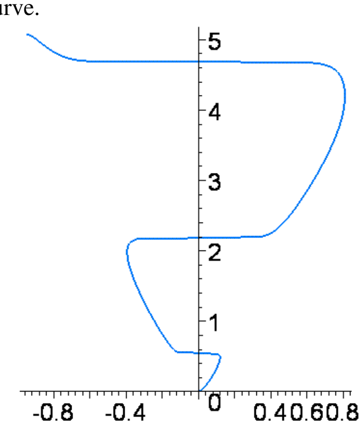
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2162893788 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3950573546 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 17.84892128.
The 2-norm of the linking coefficients is: 26.60301139.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.
The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6221, 0]$ and $[-3.5835, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the union of intervals: $[0, 0.5465] \cup [2.1841, 4.6856]$.

The coefficients in exact form are:
c[2]=1/12,
c[3]=4/27,
c[4]=2/9,
c[5]=5/9,
c[6]=2/3,
c[7]=1/6,
c[8]=4/9,
c[9]=3/4,
c[10]=11/12,
c[11]=1,
c[12]=1,

a[2,1]=1/12,
a[3,1]=4/243,
a[3,2]=32/243,
a[4,1]=1/18,
a[4,2]=0,
a[4,3]=1/6,
a[5,1]=5/9,
a[5,2]=0,
a[5,3]=-25/12,
a[5,4]=25/12,
a[6,1]=1/15,
a[6,2]=0,
a[6,3]=0,
a[6,4]=1/3,
a[6,5]=4/15,
a[7,1]=319/3840,
a[7,2]=0,
a[7,3]=0,
a[7,4]=161/1536,
a[7,5]=-41/960,
a[7,6]=11/512,
a[8,1]=245/5184,
a[8,2]=0,
a[8,3]=0,
a[8,4]=1627/10368,
a[8,5]=151/1296,
a[8,6]=-445/10368,
a[8,7]=1/6,
a[9,1]=-556349853/7539261440,
a[9,2]=0,
a[9,3]=0,
a[9,4]=-4356175383/3015704576,
a[9,5]=-814787343/1884815360,
a[9,6]=831004641/3015704576,
a[9,7]=355452237/235601920,
a[9,8]=107943759/117800960,
a[10,1]=-68998698967/1063035863040,
a[10,2]=0,
a[10,3]=0,
a[10,4]=-767387292485/425214345216,
a[10,5]=-205995991597/265758965760,
a[10,6]=-22181208863/141738115072,
a[10,7]=26226796959/15502606336,
a[10,8]=1614200643/1107329024,
a[10,9]=187/329,

a[11,1]=24511479161/17979371520,
a[11,2]=0,
a[11,3]=0,
a[11,4]=3889847115/217931776,
a[11,5]=22028391/3681280,
a[11,6]=614528179/217931776,
a[11,7]=-148401247/10215552,
a[11,8]=-3122234829/318384704,
a[11,9]=-4160/1221,
a[11,10]=15040/20757,
a[12,1]=5519/110880,
a[12,2]=0,
a[12,3]=0,
a[12,4]=0,
a[12,5]=0,
a[12,6]=83/560,
a[12,7]=932/3675,
a[12,8]=282123/1047200,
a[12,9]=2624/24255,
a[12,10]=3008/19635,
a[12,11]=37/2100,

b[1]=5519/110880,
b[2]=0,
b[3]=0,
b[4]=0,
b[5]=0,
b[6]=83/560,
b[7]=932/3675,
b[8]=282123/1047200,
b[9]=2624/24255,
b[10]=3008/19635,
b[11]=37/2100,

b*[1]=15509/341880,
b*[2]=0,
b*[3]=0,
b*[4]=0,
b*[5]=0,
b*[6]=6827/15540,
b*[7]=22138/81585,
b*[8]=78003/387464,
b*[9]=-64144/299145,
b*[10]=623408/2179485,
b*[11]=0,
b*[12]=-15/518.

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