

Verner's "most efficient" 10 stage combined order 6 and 7 Runge-Kutta scheme

The following ("most efficient") scheme comes from Jim Verner's website: <http://www.math.sfu.ca/~jverner/>.

See: J.H. Verner, SIAM Journal of Numerical Analysis 1978, 772-790, "Explicit Runge-Kutta methods with estimates of the Local Truncation Error."

The nodes of the scheme are:

$$c_2 = \frac{1}{200}, c_3 = \frac{1633}{15000}, c_4 = \frac{1633}{10000}, c_5 = \frac{911}{2000}, c_6 = \frac{3872020203200}{6348224000949}, c_7 = \frac{221}{250}, c_8 = \frac{37}{40}, c_9 = 1, c_{10} = 1.$$

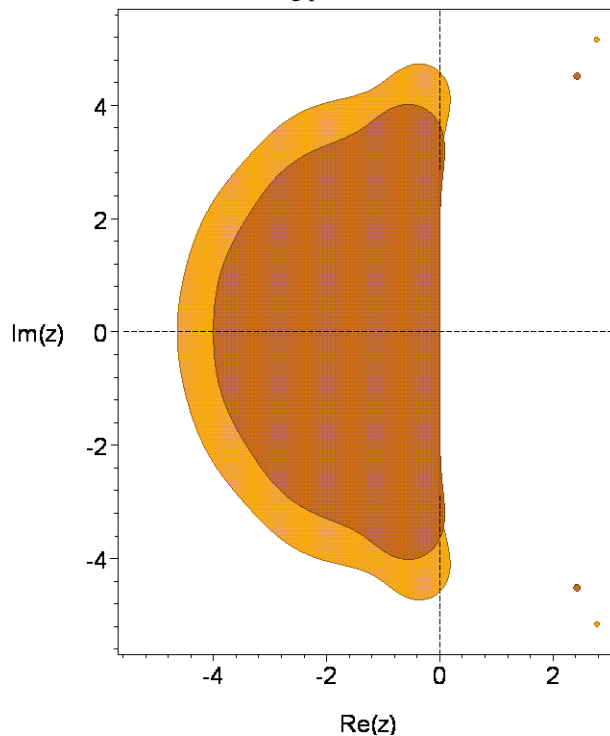
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2043042248 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3360915091 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 187.2321332.

The 2-norm of the linking coefficients is: 264.6559581.

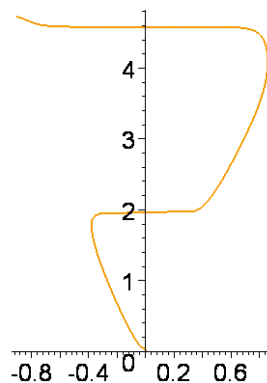
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.6408, 0]$ and $[-4.0015, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[1.9601, 4.5851]$.

The coefficients in exact form are:

$c[2]=1/200,$
 $c[3]=1633/15000,$
 $c[4]=1633/10000,$
 $c[5]=911/2000,$
 $c[6]=3872020203200/6348224000949,$
 $c[7]=221/250,$
 $c[8]=37/40,$
 $c[9]=1,$
 $c[10]=1,$

$a[2,1]=1/200,$
 $a[3,1]=-2421739/2250000,$
 $a[3,2]=2666689/2250000,$
 $a[4,1]=1633/40000,$
 $a[4,2]=0,$
 $a[4,3]=4899/40000,$
 $a[5,1]=13638791441/21333512000,$
 $a[5,2]=0,$
 $a[5,3]=-10484391993/4266702400,$
 $a[5,4]=1212514581/533337800,$
 $a[6,1]=-10566420537453573046911093467384714791794598586757942301092800/$
 $3945478828013189718441489268080494867217306323934034936408479,$
 $a[6,2]=0,$
 $a[6,3]=5231021510299954960251585060495720489874110055798400000000/$
 $481214639347870437668189933904195007588401795820714103721,$
 $a[6,4]=-53191822847901986858463766895922715840689391279600297600000000/$
 $6327491292785148384899029440906260154779895213246569749827429,$
 $a[6,5]=1781287527622331824616139968260471142273403953102720000000/$
 $2161611109404685052753814119556349840946763773078759856171,$
 $a[7,1]=40316732614812499600926954604381622079953/6622027015251398289270845012723400000000,$
 $a[7,2]=0,$
 $a[7,3]=-3102627096349187411472/125153701152335532175,$
 $a[7,4]=24512593611811814197027847310011747907776/1196407956242848578947668299729517712775,$
 $a[7,5]=-27992220377104300725776520574439264/14697934672024287924310069954740819,$
 $a[7,6]=3894588020195704963647351313475389164102598021642890694420881/$
 $3885780076153082703909872726256212057602657456212198200000000,$
 $a[8,1]=219473830564834473126851658008599711669913643457/18148581918776897851104105262715070449854720000,$
 $a[8,2]=0,$
 $a[8,3]=-10861924990722808125/217658610699713969,$
 $a[8,4]=119480318847905366486490498699126516657779132592500/2895978969529640099701689414747730700037272591519,$
 $a[8,5]=-1668548580652290279936994936449972411950403625/374242591559339035424931213674065468041890542,$
 $a[8,6]=38257721102788015064997204763622283132936825913246654962493266309964200473188255317/$
 $18800609083266437975749341539616748050410201552864110184725432140737391745319680000,$
 $a[8,7]=-500096105036391897537714558203625/5080166748283963062825254164179392,$
 $a[9,1]=7308875886605608581823594159257042017139758291/7204156842026682625849538837116971615241846400,$
 $a[9,2]=0,$
 $a[9,3]=-69262713771106609055901000000/1623478443813548051039428229,$
 $a[9,4]=14675490755672909998080009537477454813814604865188000000/$
 $410204279941884058406992680456716961361878302419014157,$
 $a[9,5]=-56001310541527335660935463926718909867728400000/12914842534226649788973963163793850259828428307,$
 $a[9,6]=1658790111713212343550432782788025386861251060331884987447839679378079088262228191320420318059/$
 $829167683860659102329644660243394316974911850817625934703082071608371922272124378570833366400,$
 $a[9,7]=397383910096482616626611556102242919581343750/1139945435719262247312274838475128039778393907,$
 $a[9,8]=-15924946453851683996573317555310225254108800/58718594835565436263767442911746035434825067,$
 $a[10,1]=-14946539621537434020656235571945101642930646111/332010640625300658857148487489318781645129600,$
 $a[10,2]=0,$
 $a[10,3]=171118989155232139053000000/913940284754207909241943,$

a[10,4]=-250906345110237533448815794107434670644623116000000/1630126656666301353726306757388291345384354510761,
a[10,5]=100053022832376562190601031855550739901200000/5412177019773639317674437354179369686847329,
a[10,6]=-741124542370420850881498253695230801778840260257881176783159378749906762930019/
104572466684527096537276822312914502674692290031200239724554350786783199401600,
a[10,7]=61868781182570033929134892454042771640656250/47410089451637179941093334205322110905900061,
a[10,8]=0,
a[10,9]=0,

b[1]=23316791871424559928103/494567385514963690893600,
b[2]=0,
b[3]=0,
b[4]=16798589731064919450625000000000000/652486332022351662028939973252423307,
b[5]=114639351263178306680000000000/4366210223630112740458758316827,
b[6]=896878436896140749138361006566548336071636396490005664374818159563751598674344336941/
5901910841358241153066354693443102222070394003395707585243672949102475401519736720800,
b[7]=7313950190577733068066406250/14824175638294700286557272779,
b[8]=-16063489150654390383296000/54687057461871573111333849,
b[9]=608799317735794481861/7494797304744605111718,
b[10]=0,

b*[1]=1774600368527162619/39781803854163746050,
b*[2]=0,
b*[3]=0,
b*[4]=326826642960527327500000000000/12237407528692429753538888074653,
b*[5]=1613021830993125656000000000/7306908939590938710641463789,
b*[6]=791180295543850558970031000997303838661625778390906982427125664631928/
3622667153393802697274794410484953168258089439622231002370760754183025,
b*[7]=3939837293445123160156250/17217393308123926000647239,
b*[8]=0,
b*[9]=0,
b*[10]=79854863595916300271/3925846207247174106138.

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