

Enright-Verner 10 stage combined order 6 and 7 Runge-Kutta scheme

See: The Relative Efficiency of Alternative Defect Control Schemes for High-Order Continuous Runge-Kutta Formulas
 W. H. Enright SIAM Journal on Numerical Analysis, Vol. 30, No. 5. (Oct., 1993), pp. 1419-1445.

The nodes of the scheme are:

$$c_2 = \frac{1}{18}, c_3 = \frac{1}{9}, c_4 = \frac{1}{6}, c_5 = \frac{4}{9}, c_6 = \frac{19}{39}, c_7 = \frac{7}{9}, c_8 = \frac{8}{9}, c_9 = 1, c_{10} = 1.$$

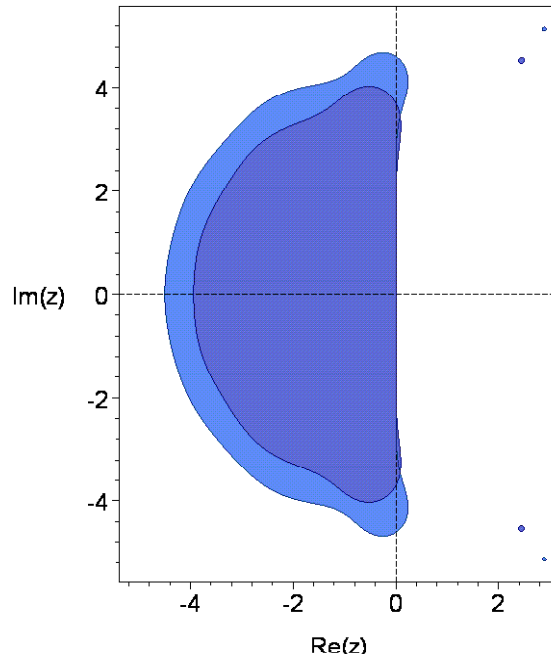
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2834216102 \times 10^{(-4)}$.

The principal error norm of the order 6 embedded scheme is: $0.3895465771 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 15.74002954.

The 2-norm of the linking coefficients is: 39.74195140.

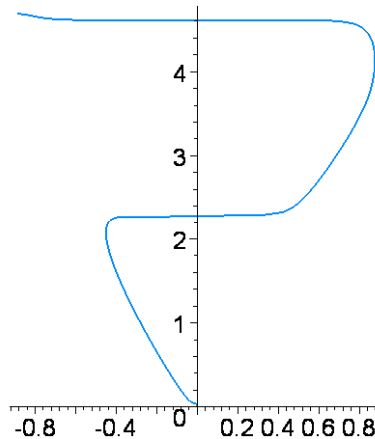
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 6 scheme appears in the darker shade.

The real stability intervals of the order 7 and 6 schemes are respectively $[-4.49987, 0]$ and $[-3.93715, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 7 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[2.2926, 4.6119]$.

The coefficients in exact form are:

$c[2]=1/18,$
 $c[3]=1/9,$
 $c[4]=1/6,$
 $c[5]=4/9,$
 $c[6]=19/39,$
 $c[7]=7/9,$
 $c[8]=8/9,$
 $c[9]=1,$
 $c[10]=1,$

$a[2,1]=1/18,$
 $a[3,1]=0,$
 $a[3,2]=1/9,$
 $a[4,1]=1/24,$
 $a[4,2]=0,$
 $a[4,3]=1/8,$
 $a[5,1]=44/81,$
 $a[5,2]=0,$
 $a[5,3]=-56/27,$
 $a[5,4]=160/81,$
 $a[6,1]=91561/685464,$
 $a[6,2]=0,$
 $a[6,3]=-12008/28561,$
 $a[6,4]=55100/85683,$
 $a[6,5]=29925/228488,$
 $a[7,1]=-1873585/1317384,$
 $a[7,2]=0,$
 $a[7,3]=15680/2889,$
 $a[7,4]=-4003076/1083375,$
 $a[7,5]=-43813/21400,$
 $a[7,6]=5751746/2287125,$
 $a[8,1]=50383360/12679821,$
 $a[8,2]=0,$
 $a[8,3]=-39440/2889,$
 $a[8,4]=1258442432/131088375,$
 $a[8,5]=222872/29425,$
 $a[8,6]=-9203268152/1283077125,$
 $a[8,7]=24440/43197,$
 $a[9,1]=-22942833/6327608,$
 $a[9,2]=0,$
 $a[9,3]=71784/5947,$
 $a[9,4]=-572980/77311,$
 $a[9,5]=-444645/47576,$
 $a[9,6]=846789710/90281407,$
 $a[9,7]=-240750/707693,$
 $a[9,8]=3972375/14534468,$
 $a[10,1]=3379947/720328,$
 $a[10,2]=0,$
 $a[10,3]=-10656/677,$
 $a[10,4]=78284/7447,$
 $a[10,5]=71865/5416,$
 $a[10,6]=-2803372/218671,$
 $a[10,7]=963000/886193,$
 $a[10,8]=0,$
 $a[10,9]=0,$

b[1]=28781/595840,
b[2]=0,
b[3]=0,
b[4]=820752/3128125,
b[5]=11259/280000,
b[6]=188245551/625100000,
b[7]=8667/43120,
b[8]=286011/2737280,
b[9]=5947/140000,
b[10]=0,

b*[1] = 577/10640,
b*[2]=0,
b*[3]=0,
b*[4]=8088/34375,
b*[5]=3807/10000,
b*[6]=-1113879/16150000,
b*[7]=8667/26180,
b*[8]=0,
b*[9]=0,
b*[10]=677/10000.

Version: 8 Oct 2011, Peter Stone