A 7 stage, combined order 6 and 4 Runge-Kutta scheme

See: A general four-parameter non-FSAL embedded Runge–Kutta algorithm of orders 6 and 4 in seven stages,
by M.E.A. El-Mikkawy and M.M.M. Eisa,
Applied Mathematics and Computation, Vol. 143, No. 2, (2003) pages 259 to 267.

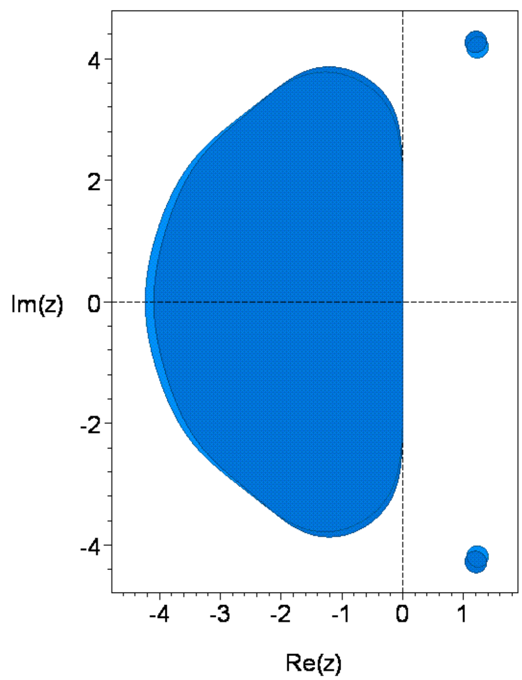
The scheme considered here can be constructed by the algorithm described in the preceding paper (when an error is corrected).
The nodes of the scheme are:

$$c_2 = \frac{4}{33},\ c_3 = \frac{2}{11},\ c_4 = \frac{11}{41},\ c_5 = \frac{18}{29},\ c_6 = \frac{10}{13},\ c_7 = 1.$$

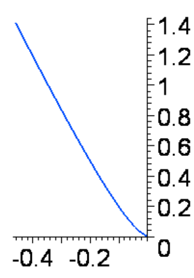
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2994603076 \times 10^{(-3)}$.

The principal error norm of the order 5 embedded scheme is: $0.7089317545 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 3.965975748.
The 2-norm of the linking coefficients is: 6.508899575.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.2342, 0]$ and $[-4.0939, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis only at the origin.

The coefficients are:
c[2]=4/33,
c[3]=2/11,
c[4]=11/41,
c[5]=18/29,
c[6]=10/13,
c[7]=1,

a[2,1]=4/33,
a[3,1]=1/22,
a[3,2]=3/22,
a[4,1]=9031/137842,
a[4,2]=3993/275684,
a[4,3]=51909/275684,
a[5,1]=8917149/16633298,
a[5,2]=-486783/1512118,
a[5,3]=-13933150/9828767,
a[5,4]=197214920/108116437,
a[6,1]=-429198695/1032365906,
a[6,2]=34485/136214,
a[6,3]=1084049890/610034399,
a[6,4]=-3723606432280/2811648544991,
a[6,5]=303886940/634254127,
a[7,1]=74753/82280,
a[7,2]=-33/136,
a[7,3]=-12628869/3465280,
a[7,4]=15819462345/3988794524,
a[7,5]=-1957436751/2457150080,
a[7,6]=422331507/521924480,

b[1]=18839/237600,
b[2]=0,
b[3]=9502009/495331200,
b[4]=810993407/2159708265,
b[5]=635845619/3547756800,
b[6]=610034399/2260742400,
b[7]=187/2430,

b*[1]=6675937/156602160,
b*[2]=0,
b*[3]=1786485503/7773161760,
b*[4]=35622025613/213252991380,
b*[5]=19967835247/83511660960,
b*[6]=11/45,
b*[7]=187/2430
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