A 7 stage, order 6 Runge-Kutta scheme with an 8 stage, order 5 embedded scheme

The node $c_3 = \dfrac{1}{7}$ has been chosen to give a large stability region.

In order to have the principal error norm a minimum with respect to the three parameters c_2, c_5 and c_6 we would need to allow c_2 to be negative which may be regarded as being undesirable.

With the arbitrary choice $c_2 = \dfrac{1}{250}$ the principal error norm is minimized with respect to the two variables c_5 and c_6.

The nodes of the scheme are:

$$c_2 = \frac{1}{250},\ c_3 = \frac{1}{7},\ c_4 = \frac{7}{43},\ c_5 = \frac{95}{158},\ c_6 = \frac{258}{341},\ c_7 = 1,\ c_8 = 1.$$

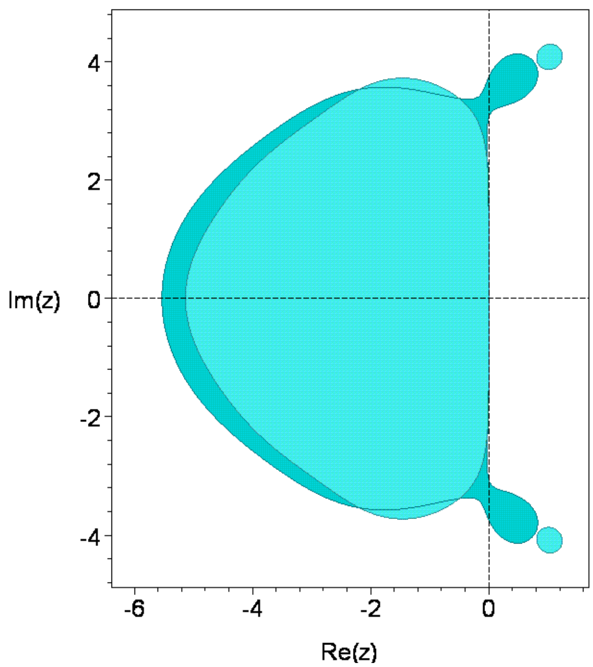
The principal error norm, that is, the 2-norm of the principal error terms is: $0.4499572779 \times 10^{(-3)}$.

The principal error norm of the order 5 embedded scheme is: $0.1469504998 \times 10^{(-2)}$.

The maximum magnitude of the linking coefficients is: $\dfrac{19495695057741}{684183816550} \simeq 28.49482052$.

The 2-norm of the linking coefficients is: 62.44686016.

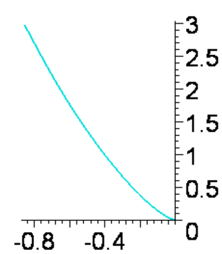
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively $[-5.1350, 0]$ and $[-5.5334, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis only at the origin.

The coefficients are:
c[2]=1/250,
c[3]=1/7,
c[4]=7/43,
c[5]=95/158,
c[6]=258/341,
c[7]=1,
c[8]=1,

a[2,1]=1/250,
a[3,1]=-118/49,
a[3,2]=125/49,
a[4,1]=-178990/79507,
a[4,2]=189875/79507,
a[4,3]=2058/79507,
a[5,1]=183462835/27610184,
a[5,2]=-21704375/3944312,
a[5,3]=-230170395/15777248,
a[5,4]=1551468165/110440736,
a[6,1]=-3054963672019106/785869789524281,
a[6,2]=2079383250/753384599,
a[6,3]=77019113540232535/5991518545696398,
a[6,4]=-2829860850134802845/246432221367666642,
a[6,5]=28969395711187248832/56520990200826970533,
a[7,1]=6974546711/2079481075,
a[7,2]=-277375/276343,
a[7,3]=-19495695057741/684183816550,
a[7,4]=1509943351361811/55749765668170,
a[7,5]=-2499228685418976/3234273504085325,
a[7,6]=5283402594971741163/6342590386340363275,
a[8,1]=-80385864333218/20960037611415,
a[8,2]=799807750/456794979,
a[8,3]=1803928451372705/63920235001428,
a[8,4]=-23135213712637169/876350204132004,
a[8,5]=211421687651408296/167497649134991955,
a[8,6]=0,
a[8,7]=0,

b[1]=73939/541800,
b[2]=0,
b[3]=-1504848359/802175400,
b[4]=320331397297/147069327510,
b[5]=11988211730504/59724681614325,
b[6]=38283085461111403/137160413503399800,
b[7]=276343/3388392,
b[8]=0,

b*[1]=555367/4738600,
b*[2]=0,
b*[3]=-12245001559/7754362200,
b*[4]=102355483139/54158863464,
b*[5]=119074760026072/522353961420525,
b*[6]=1382103753820537/5324835330118600,
b*[7]=1/29,
b*[8]=1/20.

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