A 7 stage, order 6 Runge-Kutta scheme with an 8 stage, order 5 embedded scheme

See: On the Optimization of Some Seven-stage Sixth-order Runge-Kutta Methods, by M. Tanaka, K. Kasuga, S. Yamashita and H. Yazaki, Journal of the Information Processing Society of Japan, Vol. 33, No. 8 (1992), pages 993 to 1005.

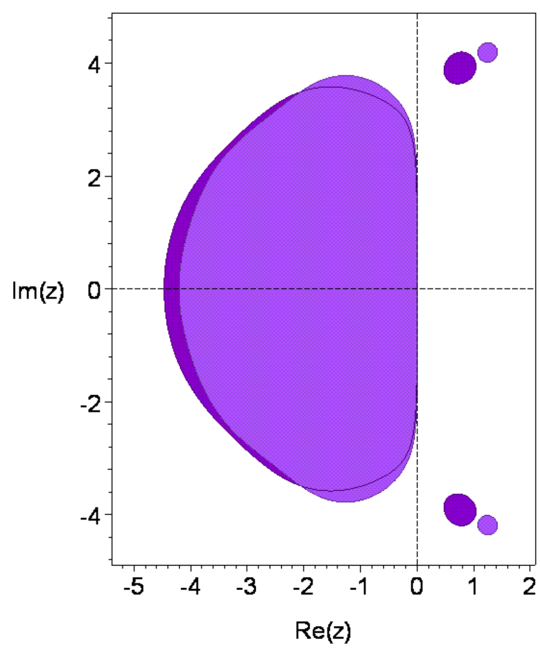
The scheme considered here is essentially the same as a scheme of Tanaka, Yamashita et. al., but the parameters are changed slightly.
In particular, the nodes c_3 and c_4 are given in exact radical form so that the stability function coincides exactly with that of a scheme of John Butcher.
The nodes of the scheme are:

$$c_2 = \frac{72}{2129},\ c_3 = \frac{1}{3} - \frac{\sqrt{5}}{15},\ c_4 = \frac{1}{2} - \frac{\sqrt{5}}{10},\ c_5 = \frac{164}{267},\ c_6 = \frac{373}{486},\ c_7 = 1,\ c_8 = 1.$$

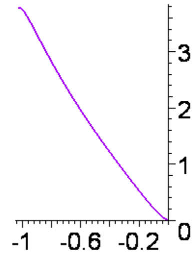
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2867458817 \times 10^{(-3)}$.
Note: 5 of the 48 principal error conditions are satisfied by the order 6 scheme.

The principal error norm of the order 5 embedded scheme is: $0.9317558375 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 7.157182281.
The 2-norm of the linking coefficients is: 12.14569603.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.2063, 0]$ and $[-4.46765, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis only at the origin.

The coefficients are:

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c[2]=72/2129,
c[3]=1/3-1/15*5^(1/2),
c[4]=1/2-1/10*5^(1/2),
c[5]=164/267,
c[6]=373/486,
c[7]=1,
c[8]=1,

a[2,1]=72/2129,
a[3,1]=-1769/1080+1913/3240*5^(1/2),
a[3,2]=2129/1080-2129/3240*5^(1/2),
a[4,1]=1/8-1/40*5^(1/2),
a[4,2]=0,
a[4,3]=3/8-3/40*5^(1/2),
a[5,1]=-802218259/1142049780+213729023/228409956*5^(1/2),
a[5,2]=1149683419/1142049780-1149683419/1142049780*5^(1/2),
a[5,3]=-1025410/2114907-580232/2114907*5^(1/2),
a[5,4]=1681000/2114907+10954544/31723605*5^(1/2),
a[6,1]=7575741870545309/11157937369886880-13418350043007617/17852699791819008*5^(1/2),
a[6,2]=-20211071767/27549901440+20211071767/27549901440*5^(1/2),
a[6,3]=800522380228075315/733121842670551296+124854653494021519/733121842670551296*5^(1/2),
a[6,4]=-65020768543225/108857925559872-119308096722133/544289627799360*5^(1/2),
a[6,5]=984441941431614994 3/300579955494926031 36+3141622173139307 9/469656180460821924*5^(1/2),
a[7,1]=104312801370347863/11510530743839160-12890225093922337/3836843581279720*5^(1/2),
a[7,2]=-7075270633241/752666628120+474513663903/125444438020*5^(1/2),
a[7,3]=4709287773490366905/2488364572604448124 4-3972399616956725282 7/24883645726044481244*5^(1/2),
a[7,4]=35373605834732031621/42050872469651300 42-43120732323530496113/21025436234825650210*5^(1/2),
a[7,5]=-380645474343673404550 95/10309953901030972773179+
       1028294505461819196086 31/82479631208247782185432*5^(1/2),
a[7,6]=-27680991515034913595197 5624/7750654030292046570557 8687+
       15326758166386626571811 4072/775065403029204657055 78687*5^(1/2),
a[8,1]=-2208473159044618426 37/3414165371548758288 0+1260180975895098530 5/6828330743097516576*5^(1/2),
a[8,2]=198904558570001/31611998381040-72058899075311/31611998381040*5^(1/2),
a[8,3]=19685893447444038465/3338162851663332968+940478243235908539/1669081425831666484*5^(1/2),
a[8,4]=-627187437430245/1156560085212994-233677195819899 98/8674200639097455*5^(1/2),
a[8,5]=-1601264984412197344 6539/383221095370950624 7264+982741361944139727 4227/383221095370950624 7264*5^(1/2),
a[8,6]=0,
a[8,7]=0,

b[1]=1693/17904-11017/1468128*5^(1/2),
b[2]=0,
b[3]=839198240145/603026201888-361410047595/603026201888*5^(1/2),
b[4]=52552674535/8045162904-7419008745/2681720968*5^(1/2),
b[5]=-730277710680771/207650652577952+490939506158965 47/29797868644936112*5^(1/2),
b[6]=-728135604777296052/199479072567197533+245468610662923710 0/139635350797038273 1*5^(1/2),
b[7]=788839/5307384-170633/5307384*5^(1/2),
b[8]=0,

b*[1]=3080373/34256320-97833/6851264*5^(1/2),
b*[2]=0,
b*[3]=119740465905/111083774032-5524275213/13885471754*5^(1/2),
b*[4]=6380291900/2346505847-1516138407/1340860484*5^(1/2),
b*[5]=-1504364250006177/826570558805440+7525184485117449/8100391476293312*5^(1/2),
b*[6]=-71215182039129174/61785553450016935+53098363236260610/86499774830023709*5^(1/2),
b*[7]=1/28,
b*[8]=1/20.
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