An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

See: Global Error estimation with Runge-Kutta triples, by J.R.Dormand, M.A.Lockyer, N.E.McCorrigan and P.J.Prince,
Computers and Mathematics with Applications, 18 (1989) pages 835-846.

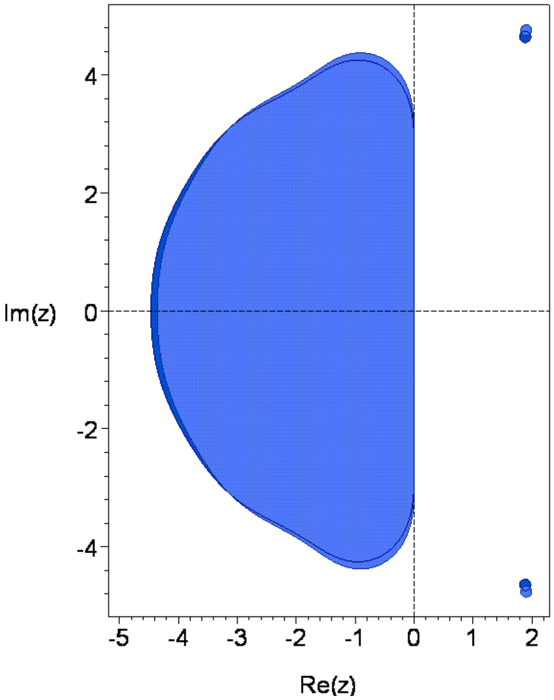
The scheme considered here is constructed in the manner of the schemes in the preceding paper.
The nodes of the scheme are:

$$c_2 = \frac{14}{135},\ c_3 = \frac{7}{45},\ c_4 = \frac{7}{30},\ c_5 = \frac{51612}{89803},\ c_6 = \frac{49}{87},\ c_7 = \frac{81}{82},\ c_8 = 1,\ c_9 = 1.$$

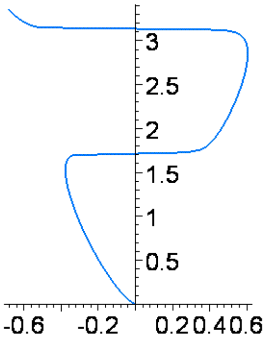
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2240027910 \times 10^{(-4)}$.
Note: The scheme satisfies 18 of the 48 principal error conditions including the order 7 quadrature condition.

The principal error norm of the order 5 embedded scheme is: $0.1044136456 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 26.31173083.
The 2-norm of the linking coefficients is: 49.12685461.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.3579, 0]$ and $[-4.4659, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[1.7253, 3.1308]$.

The coefficients are:
c[2]=14/135,
c[3]=7/45,
c[4]=7/30,
c[5]=51612/89803,
c[6]=49/87,
c[7]=81/82,
c[8]=1,
c[9]=1,

a[2,1]=14/135,
a[3,1]=7/180,
a[3,2]=7/60,
a[4,1]=7/120,
a[4,2]=0,
a[4,3]=7/40,
a[5,1]=19468520817039492/35486945168446723,
a[5,2]=0,
a[5,3]=-72573280055824680/35486945168446723,
a[5,4]=73499982271800480/35486945168446723,
a[6,1]=20252658340120203773069749/33784135725639504675502296,
a[6,2]=0,
a[6,3]=-79221600091287928 0500/3515332122172081842 83,
a[6,4]=224903465917229545508505100/100340318815275363580633077,
a[6,5]=-21679358721707162238705728210467/90110502903675461933151238040 1576,
a[7,1]=-97215872112419344081463091846 7125/799434474429485902833274628345432,
a[7,2]=0,
a[7,3]=3058612407219548732633706208815/7093473597422235162673244 26216,
a[7,4]=-53478378574860120460721550328853039040/23405367722957560812286413128409759761,
a[7,5]=524767583662593596626716546857270370413091/24088433982273003693314369668047287199799,
a[7,6]=-69620601045863376441 4572/32219569565392536806647,
a[8,1]=-1143619992049336937444337215079 07/7787742775937515783237730500 6776,
a[8,2]=0,
a[8,3]=74051210740135739810 98320/14573249381940422602 33261,
a[8,4]=-2993009538247695784606055118276 4365/11155804472406678900783685305189917,
a[8,5]=264991972525156064142611927680686033758119 9555/10071248227801273316057581503950716982 6241912,
a[8,6]=-42066702439513241792550/16040076479630245 81459,
a[8,7]=-42534274123407403805 9/27197351224216978505337,
a[9,1]=5847536753/86036171760,
a[9,2]=0,
a[9,3]=0,
a[9,4]=1815811171875/4929812076868,
a[9,5]=749284986843781886973742184 47/297448545566487493703246 59440,
a[9,6]=-6206299092285813/2910288487182440,
a[9,7]=1044913680622457/1298591690996340,
a[9,8]=-418874561/667578680,

b[1]=5847536753/86036171760,
b[2]=0,
b[3]=0,
b[4]=1815811171875/4929812076868,
b[5]=74928498684378188697374218447/29744854556648749370324659440,
b[6]=-6206299092285813/2910288487182440,
b[7]=1044913680622457/1298591690996340,
b[8]=-418874561/667578680,

b*[1]=13608333502668105982729889189/197441635892684338840372330104,
b*[2]=0,
b*[3]=0,
b*[4]=7154586848463452050019495500125/19675246518315548228466935287856,
b*[5]=85316751507451678656433588557034105747933/36476436347097667766853358930957710182664,
b*[6]=-6519743523284530424551776811925577/3339363595999139623414387108668538,
b*[7]=20251703785102111771440737597924269/23840769535138099800229313089019088,
b*[8]=-25/38,
b*[9]=-245172123941262584/22948677498512207029.

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