An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

See: Global Error estimation with Runge-Kutta triples, by J.R.Dormand, M.A.Lockyer, N.E.McCorrigan and P.J.Prince, Computers and Mathematics with Applications, 18 (1989) pages 835-846.

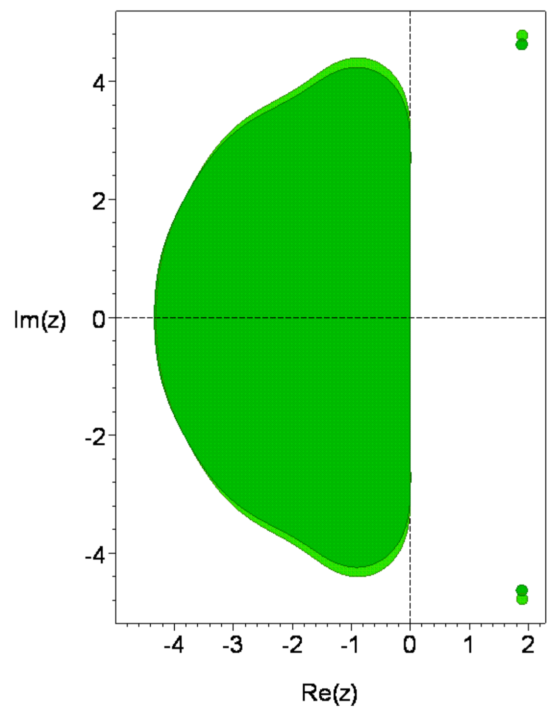
The scheme considered here is constructed in the manner of the schemes in the preceding paper.
The nodes of the scheme are:

$$c_2=\frac{8}{75},\ c_3=\frac{4}{25},\ c_4=\frac{6}{25},\ c_5=\frac{1807}{3162},\ c_6=\frac{11}{21},\ c_7=\frac{63}{64},\ c_8=1,\ c_9=1.$$

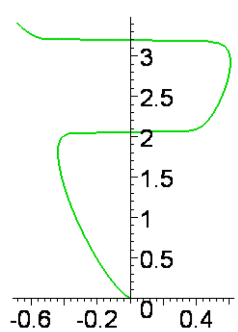
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2886234837 \times 10^{(-4)}$.
Note: The scheme satisfies 18 of the 48 principal error conditions including the order 7 quadrature condition.

The principal error norm of the order 5 embedded scheme is: $0.1274365644 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 9.177199352.
The 2-norm of the linking coefficients is: 18.75931408.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.3080, 0]$ and $[-4.3378, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[2.0579, 3.1899]$.

The coefficients are:
c[2]=8/75,
c[3]=4/25,
c[4]=6/25,
c[5]=1807/3162,
c[6]=11/21,
c[7]=63/64,
c[8]=1,
c[9]=1,

a[2,1]=8/75,
a[3,1]=1/25,
a[3,2]=3/25,
a[4,1]=3/50,
a[4,2]=0,
a[4,3]=9/50,
a[5,1]=1116733125001/2276240222016,
a[5,2]=0,
a[5,3]=-1364629188325/758746740672,
a[5,4]=2138982988675/1138120111008,
a[6,1]=37318488313171592359/62942867997011834504 4,
a[6,2]=0,
a[6,3]=-128555397896225575/58054665188168082,
a[6,4]=2022450263344324777397 5/912723835155340951587 6,
a[6,5]=-35907783794909599007 60/50904073152027811713543,
a[7,1]=-9955638050099787690037550073/7170610831257903919821488128,
a[7,2]=0,
a[7,3]=4217703686179287684108 75/90187287207047138902016,
a[7,4]=-30319799014068178151708710638 75/140845378206500868366711704780 8,
a[7,5]=15692549351917507935540245739734013/22525630564781892444737725949542 40,
a[7,6]=-1487354013567533229 3/208966554719617024 0,
a[8,1]=-26917782864536563802453 83/15212581634849805747351 72,
a[8,2]=0,
a[8,3]=2141335633391996675/371194659287217497,
a[8,4]=-161679233545622750791763569225/621374270836077357030944268 84,
a[8,5]=1363639349516531201308323118018680/15493565926242760408341489041765 3,
a[8,6]=-444161905271490/48398415269957,
a[8,7]=-311127404078694400/144775150228556833 23,
a[9,1]=31138111/450810360,
a[9,2]=0,
a[9,3]=0,
a[9,4]=2474072265625/6361142786136,
a[9,5]=138815735069536848 72/141393345141145223 75,
a[9,6]=-132955185321/214068151000,
a[9,7]=19335942766592/29106652419495,
a[9,8]=-2486977/5149000,

b[1]=31138111/450810360,
b[2]=0,
b[3]=0,
b[4]=2474072265625/6361142786136,
b[5]=13881573506953684872/14139334514114522375,
b[6]=-132955185321/214068151000,
b[7]=19335942766592/29106652419495,
b[8]=-2486977/5149000,

b*[1]=2435936702453593627462l/339039904141177334687592,
b*[2]=0,
b*[3]=0,
b*[4]=470295508478676388272578125/1258949982543885938072825784,
b*[5]=6740104862258656670972654206056/7847776077910531141899035067695,
b*[6]=-15929717413567933460447487/32198747781536819606222440,
b*[7] = 16516196509757640754673483776/21890172724016706578665261989,
b*[8]=-6/11,
b*[9]=-559011373156103/30082707428567288.

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