An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

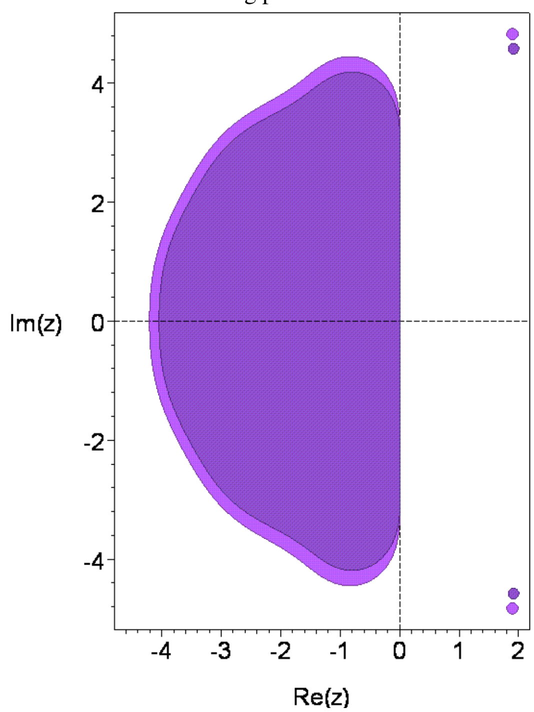
See: Global Error estimation with Runge-Kutta triples, by J.R.Dormand, M.A.Lockyer, N.E.McCorrigan and P.J.Prince, Computers and Mathematics with Applications, 18 (1989) pages 835-846.

The nodes of the scheme are:

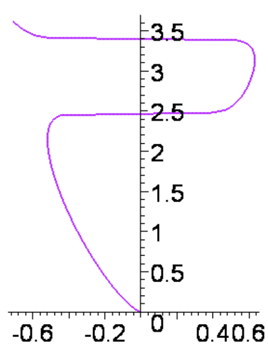
$$c_2 = \frac{1}{9},\ c_3 = \frac{1}{6},\ c_4 = \frac{1}{4},\ c_5 = \frac{5}{9},\ c_6 = \frac{1}{2},\ c_7 = \frac{48}{49},\ c_8 = 1,\ c_9 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.4375865965 \times 10^{(-4)}$.
Note: The scheme satisfies 18 of the 48 principal error conditions including the order 7 quadrature condition.
The principal error norm of the order 5 embedded scheme is: $0.2013241697 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: $\frac{152952}{12173} \simeq 12.56485665$.
The 2-norm of the linking coefficients is: 21.08468413.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.2063, 0]$ and $[-4.0453, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[2.4752, 3.4020]$.

The Butcher tableau is as follows.

$$
\begin{bmatrix}
\frac{1}{9} & \frac{1}{9} & & & & & & & & \\
\frac{1}{6} & \frac{1}{24} & \frac{1}{8} & & & & & & & \\
\frac{1}{4} & \frac{1}{16} & 0 & \frac{3}{16} & & & & & & \\
\frac{5}{9} & \frac{280}{729} & 0 & -\frac{325}{243} & \frac{1100}{729} & & & & & \\
\frac{1}{2} & \frac{6127}{14680} & 0 & -\frac{1077}{734} & \frac{6494}{4037} & -\frac{9477}{161480} & & & & \\
\frac{48}{49} & -\frac{13426273320}{14809773769} & 0 & \frac{4192558704}{2115681967} & \frac{14334750144}{14809773769} & \frac{117092732328}{14809773769} & -\frac{361966176}{40353607} & & & \\
1 & -\frac{2340689}{1901060} & 0 & \frac{31647}{13579} & \frac{253549596}{149518369} & \frac{10559024082}{977620105} & -\frac{152952}{12173} & -\frac{5764801}{186010396} & & \\
1 & \frac{203}{2880} & 0 & 0 & \frac{30208}{70785} & \frac{177147}{164560} & -\frac{536}{705} & \frac{1977326743}{3619661760} & -\frac{259}{720} & \\
b & \frac{203}{2880} & 0 & 0 & \frac{30208}{70785} & \frac{177147}{164560} & -\frac{536}{705} & \frac{1977326743}{3619661760} & -\frac{259}{720} & \\
b* & \frac{36567}{458800} & 0 & 0 & \frac{9925984}{27063465} & \frac{85382667}{117968950} & -\frac{310378}{808635} & \frac{262119736669}{345979336560} & -\frac{1}{2} & -\frac{101}{2294}
\end{bmatrix}
$$

The coefficients are:
c[2]=1/9,
c[3]=1/6,
c[4]=1/4,
c[5]=5/9,
c[6]=1/2,
c[7]=48/49,
c[8]=1,
c[9]=1,

a[2,1]=1/9,
a[3,1]=1/24,
a[3,2]=1/8,
a[4,1]=1/16,
a[4,2]=0,
a[4,3]=3/16,
a[5,1]=280/729,
a[5,2]=0,
a[5,3]=-325/243,
a[5,4]=1100/729,
a[6,1]=6127/14680,
a[6,2]=0,
a[6,3]=-1077/734,
a[6,4]=6494/4037,
a[6,5]=-9477/161480,
a[7,1]=-13426273320/14809773769,
a[7,2]=0,
a[7,3]=4192558704/2115681967,
a[7,4]=14334750144/14809773769,
a[7,5]=117092732328/14809773769,
a[7,6]=-361966176/40353607,
a[8,1]=-2340689/1901060,
a[8,2]=0,

a[8,3]=31647/13579,
a[8,4]=253549596/149518369,
a[8,5]=10559024082/977620105,
a[8,6]=-152952/12173,
a[8,7]=-5764801/186010396,
a[9,1]=203/2880,
a[9,2]=0,
a[9,3]=0,
a[9,4]=30208/70785,
a[9,5]=177147/164560,
a[9,6]=-536/705,
a[9,7]=1977326743/3619661760,
a[9,8]=-259/720,

b[1]=203/2880,
b[2]=0,
b[3]=0,
b[4]=30208/70785,
b[5]=177147/164560,
b[6]=-536/705,
b[7]=1977326743/3619661760,
b[8]=-259/720,

b*[1]=36567/458800,
b*[2]=0,
b*[3]=0,
b*[4]=9925984/27063465,
b*[5]=85382667/117968950,
b*[6]=-310378/808635,
b*[7]=262119736669/345979336560,
b*[8]=-1/2,
b*[9]=-101/2294.

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