An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

See: A General Family of Explicit Runge-Kutta Pairs of Order 6(5), by S. N. Papakostas, Ch. Tsitouras and G. Papageorgiou, Siam Journal of Numerical Analysis, Volume 33, Number 3, June 1996, Pages 917-936.

The scheme considered here is constructed in the manner of the schemes in the preceding paper.

The nodes of the scheme are:

$$c_2 = \frac{1}{10},\ c_3 = \frac{12}{77},\ c_4 = \frac{32}{137},\ c_5 = \frac{158}{275},\ c_6 = \frac{307}{549},\ c_7 = \frac{75}{76},\ c_8 = 1,\ c_9 = 1.$$

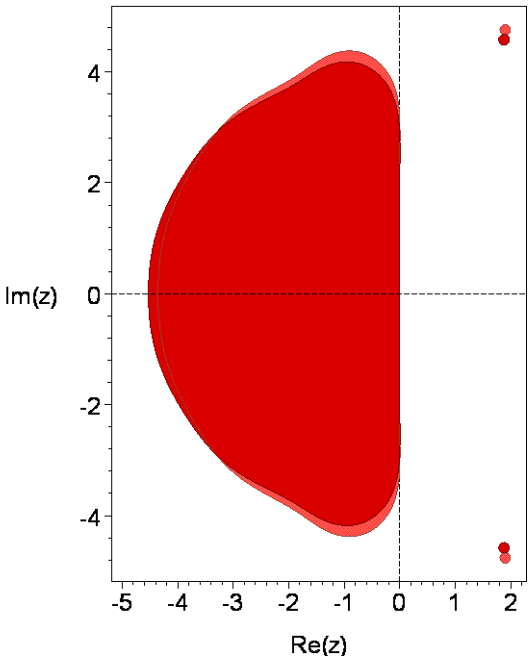
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2278113268 \times 10^{(-4)}$.

The principal error norm of the order 5 embedded scheme is: $0.1733459642 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 20.53886216.

The 2-norm of the linking coefficients is: 38.59329490.

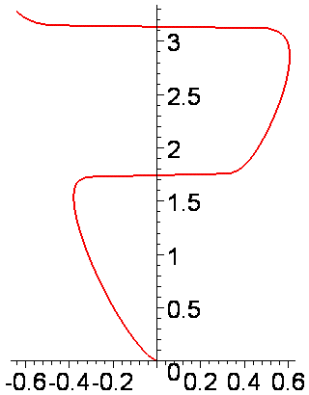
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively [−4.3540, 0] and [−4.5284, 0].

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [1.7413, 3.1386].

The coefficients correct to 85 digits are:
c[2]=.1,
c[3]=.1558441558441558441558441558441558441558441558441558441558441558441558441558441558442,
c[4]=.2335766423357664233576642335766423357664233576642335766423357664233576642335766423358,
c[5]=.5745454545454545454545454545454545454545454545454545454545454545454545454545454545455,
c[6]=.5591985428051001821493624772313296903460837887067395264116575591985428051001821493625,
c[7]=.9868421052631578947368421052631578947368421052631578947368421052631578947368421052632,
c[8]=1.,
c[9]=1.,

a[2,1]=.1,
a[3,1]=.3440715129026817338505650193961882273570585258896947208635520323832012143700455388767e-1,
a[3,2]=.1214370045538876707707876539045370214201383032551863720694889526058357227188396019565,
a[4,1]=.5829840468336233017317255848488238595703420641886889033819074151926156298951477209772e-1,
a[4,2]=.6637232097913692389441148946358849472113189696889795631586431993843967229185050308793e-3,
a[4,3]=.1746145144426127239455475601971240648621778322756757067409863817047117045211433652072,
a[5,1]=.5458355347064458345762506591746280285467602126649199490351101198699579879232136176355,
a[5,2]=-.1232427762503086638876641369882958544270091021829678068633297032019265533882720134364e-2,
a[5,3]=-2.034936183938130133829650336737449583148486470756377838760088882664006595007654805104,
a[5,4]=2.064878531539641931346821773478159058600541803658742113248157514371522418072869362148,
a[6,1]=.6101459882655427642477222681654439375584635011031527478885661220023729800211191637530,
a[6,2]=-.1439869561617104345698987626431092683794697452592292608465632591375781232822052115035e-2,
a[6,3]=-2.298505491986357524232908523827450441406002197024852311335814615119329049074338017282,
a[6,4]=2.280112907501267597261678576269440895750975513263438774095562690231935472971676957798,
a[6,5]=-.3111499141373555078143085574967360887355833118240739162819100532506081758545390279140e-1,
a[7,1]=-1.247229877860156324140566013671619045780096635647522730819215424289577253043763183702,
a[7,2]=.1293491456380863075574830883333406790441842200957566785551543795379096841115847423 20e-2,
a[7,3]=4.413269079364893627022520352984521325345322367484345689152465467627686375168977912837,
a[7,4]=-2.337705425393248035091565803468045399290322975790989609654554013622547248103870707895,
a[7,5]=16.75491507770363078604438181675655558719275700402416176114145773085411492163957874275,
a[7,6]=-16.59770024000834302217350307822158791340986183902693297176186680968605681060819224347,
a[8,1]=-1.529999864996572411040781568274812693724191020073510809873317679952856533095682434369,
a[8,2]=.1513815799428084467899678018432952462703709488370454955876406794306405765036802745539e-2,
a[8,3]=5.274723996295912471744241694067637001592600259550895522295965124971174648430209221513,
a[8,4]=-2.781335257745429345576097142864000740635803757322249110240593876053101658678433444344,
a[8,5]=20.53886216275637103792769287496559215515836526060121178015017254743766780210470079640,
a[8,6]=-20.48654067324890986952650632490394017441434344787701238352578636072669871640508186382,
a[8,7]=-.1722417886079996799644921100890850043933100436770545376231616247049194812074907813338e-1,
a[9,1]=.6819221622415060779009098056899939620655650981576540106346571931628519543481521557881e-1,
a[9,2]=0.,
a[9,3]=-.1745457122612639870357579553698611474462739863707961477717303400831146324597037465220e-2,
a[9,4]=.3714879041963906408821675856982364578374864917853302531715380057212329824544470061038,
a[9,5]=2.003096623562377135616428237123863762593627126417726966100871241092949740208404098264,
a[9,6]=-1.618802261923254216611238209259393832503967302241831130694034290710326744218543328566,
a[9,7]=.7530115893039758440980898304068240383451610596446518811554562094941345780938822940244,
a[9,8]=-.5752406142410273719051808449848312110044011455579354093195795815134446056484082479393,

b[1]=.6819221622415060779009098056899939620655650981576540106346571931628519543481521557881e-1,
b[2]=0.,
b[3]=-.1745457122612639870357579553698611474462739863707961477717303400831146324597037465220e-2,
b[4]=.3714879041963906408821675856982364578374864917853302531715380057212329824544470061038,
b[5]=2.003096623562377135616428237123863762593627126417726966100871241092949740208404098264,
b[6]=-1.618802261923254216611238209259393832503967302241831130694034290710326744218543328566,
b[7]=.7530115893039758440980898304068240383451610596446518811554562094941345780938822940244,
b[8]=-.5752406142410273719051808449848312110044011455579354093195795815134446056484082479393,

b*[1]=.69883477410669888873664789310187601622534330803775450810817935159582111032915888070 77e-1,
b*[2]=0.,
b*[3]=-.14480593011162782456064744283723667953810899345769213593527039673656534291700374284 06e-2,
b*[4]=.36257250091952532009722683362296243490435751206228204407888149552080846095836148611 36,
b*[5]=1.75847515561178275798633492200619420561953902342190470886241184131060309178021674499 0,
b*[6]=-1.37382682311477621656763721651806318879061945249104259431214601915771634638334626154 2,
b*[7]=.82752556665573270967419896418890949525775149431947549373756926931590651785920399797 70,
b*[8]=-.625,
b*[9]=-.18181818181818181818181818181818181818181818181818181818181818181818181818181818181818182e-1.

Version: 8 Nov 2011, Peter Stone