

**An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme**

See: A General Family of Explicit Runge-Kutta Pairs of Order 6(5), by S. N. Papakostas, Ch. Tsitouras and G. Papageorgiou, Siam Journal of Numerical Analysis, Volume 33, Number 3, June 1996, Pages 917-936.

The scheme considered here is the scheme with a small principal error norm as described in the preceding paper.

The nodes of the scheme are:

$$c_2 = \frac{700}{12757}, c_3 = \frac{487}{5394}, c_4 = \frac{207}{2830}, c_5 = \frac{4783}{8682}, c_6 = \frac{3736}{5943}, c_7 = \frac{6631}{6733}, c_8 = 1, c_9 = 1.$$

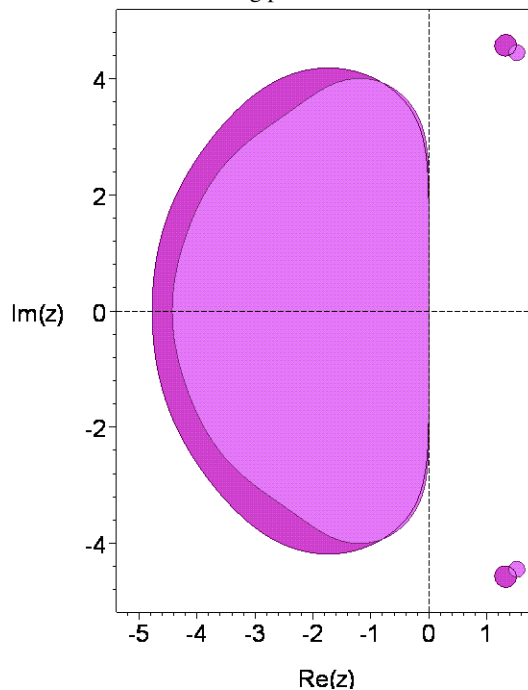
The principal error norm, that is, the 2-norm of the principal error terms is:  $0.1252244078 \times 10^{(-4)}$ .

The principal error norm of the order 5 embedded scheme is:  $0.5407168241 \times 10^{(-3)}$ .

The maximum magnitude of the linking coefficients is: 33.07623222.

The 2-norm of the linking coefficients is: 78.37863913.

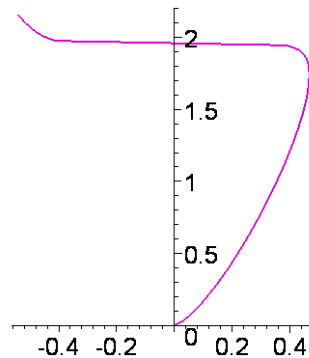
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively  $[-4.4286, 0]$  and  $[-4.7741, 0]$ .

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval:  $[0, 1.9562]$ .

The coefficients correct to 85 digits are:

c[2]=.5487183507094144391314572391628125734890648271537195265344516735909696637140393509446e-1,  
c[3]=.9028550241008527994067482387838338895068594734890619206525769373377827215424545791620e-1,  
c[4]=.7314487632508833922261484098939929328621908127208480565371024734982332155477031802120e-1,  
c[5]=.5509099285878829762727482146970744068187053674268601704676341856715042616908546417876,  
c[6]=.6286387346458017836109708901228335857311122328790173313141511021369678613494867911829,  
c[7]=.9848507351849101440665379474231397593940294073964057626615178969255903757611762958562,  
c[8]=1.,  
c[9]=1.,

a[2,1]=.5487183507094144391314572391628125734890648271537195265344516735909696637140393509446e-1,  
a[3,1]=.1600812554722999030788800907867823152510996252837700695349968241241575474025075052542e-1,  
a[3,2]=.7427737686285528963278681479970515742557598482052918511175801132136251741399470739078e-1,  
a[4,1]=.1713261303329450305960097857427853684626850986330486942023311461646544353108392481139e-1,  
a[4,2]=.6726241717096721885475578379674412542971034928638639956428965278307211072184432193336e-1,  
a[4,3]=-.1125015387917338269174192138162336898975977787760646333081252004971423269815792872355e-1,  
a[5,1]=5.297719577258082712529275039380161822174239358741716289330057058579067195510799726800,  
a[5,2]=-.4411526061912926972084043889085420792938204160244425018583903253696010551650762091522,  
a[5,3]=28.63923856646282213001664629351073851163571217597829117280674571891578917165178627232,  
a[5,4]=-32.94489560894172916906476872928528384769742575126870478981077826645375105030665514818,  
a[6,1]=-4.371677273449296236244595535672660353672217291310283169262192175635882770915239739025,  
a[6,2]=.1689383132812812674738510573402185395015020037870021605942195177966143168054289117833,  
a[6,3]=-20.19078718042904527762169916190039905424525343417614856648426201029029859585566652267,  
a[6,4]=-24.6432849578394995367124433067150112277682972938775607676252888011018078042908562954,  
a[6,5]=.3788799174033624932909712236406632263702512251906908297038568901563541308858785115510,  
a[7,1]=3.908640950601421717849771026981248918762498222747083159131417351101738337296533842639,  
a[7,2]=.8552190355493487419841797523855808144718733455505122111739814990821666458520680373560,  
a[7,3]=22.69205866067023759004936816268302870823223743770423357045970334886351242175539675846,  
a[7,4]=-26.89195193812567375928112576874044212082475163507720564635206767800115282186106850623,  
a[7,5]=-1.179725149415687186407764555446876378357149926946757148184971818819162062568528548764,  
a[7,6]=1.600609175905263039872109329560599817109321963418539616433455194698487855286774712395,  
a[8,1]=4.794880221599204973543847410662191962061501996643878416087763888199061726560874590551,  
a[8,2]=1.098425214084909931330934622756802644654019270468011686907235910368264186932203723321,  
a[8,3]=27.81613893362125450128116379106794787310125224336608813018908369335411607922027565605,  
a[8,4]=-33.07623221864911798168659941173284463263304078376322101019419871746467000441914265099,  
a[8,5]=-1.600487232181175127460931028108210878695919397149830778148816621183252920128204453211,  
a[8,6]=1.9877426180903065564870807079013913235307564786338379057986320556518189332426180118133,  
a[8,7]=-2046753656538285349549609254727829201856980820330550282738870979170840059218698385024e-1,  
a[9,1]=.3160843116756840262350221313512842809639381788258701816945400719040997102825482847084,  
a[9,2]=0.,  
a[9,3]=2.985109097250934336074996380546541800975670953247348944187721715087733831599898507172,  
a[9,4]=-2.945840660332062629483228722724796142858934897712891144248878142050445755264421064182,  
a[9,5]=.2152052807306582471976489160399563898831490656388982243908935877196163521604790079611,  
a[9,6]=.2459668995358545735557862926374984411146387949373372012801159530477447471560718335804,  
a[9,7]=.7061095979470043546044274467302751276591380010815047434236191159889710522732167881392,  
a[9,8]=-5226345268080729081846524445807598977376000960180681507280123016977199382077933573787,

b[1]=.3160843116756840262350221313512842809639381788258701816945400719040997102825482847084,  
b[2]=0.,  
b[3]=2.985109097250934336074996380546541800975670953247348944187721715087733831599898507172,  
b[4]=-2.945840660332062629483228722724796142858934897712891144248878142050445755264421064182,  
b[5]=.2152052807306582471976489160399563898831490656388982243908935877196163521604790079611,  
b[6]=.2459668995358545735557862926374984411146387949373372012801159530477447471560718335804,  
b[7]=.7061095979470043546044274467302751276591380010815047434236191159889710522732167881392,  
b[8]=-5226345268080729081846524445807598977376000960180681507280123016977199382077933573787,

b\*[1]=.1986067229089250567554539619354549032688891473101943037456391982135652974618774216133,  
b\*[2]=0.,  
b\*[3]=.1649009069929464292286508839148291972272585272437034070526651896113624034331023749428,  
b\*[4]=.6714805704943519524968190410461194750271489386836308656232219018270979200456349268161,  
b\*[5]=.2631758620767233934867111040933295447070246544613108002740522483152101594661133387776,  
b\*[6]=.2785803561447122113978100020076653403539436610559103588896145514609509857544451559736,  
b\*[7]=.3043475216034583522863366472593454259212553657546472390325270669545004725615696800119,  
b\*[8]=-.2040766804173744236835719485561756855132249454016015530211113807328318369327833050231,  
b\*[9]=.5e-1.

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