An 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

The scheme considered here has $c_6 = \frac{97}{140}$ and $c_7 = \frac{35}{36}$.

With c_6 and c_7 having these fixed values, the nodes c_2, c_4 and c_5 have been chosen to minimize the principal error norm.

The nodes of the scheme are:

$$c_2 = \frac{7}{95},\ c_3 = \frac{35}{243},\ c_4 = \frac{35}{162},\ c_5 = \frac{31}{55},\ c_6 = \frac{97}{140},\ c_7 = \frac{35}{36},\ c_8 = 1,\ c_9 = 1.$$

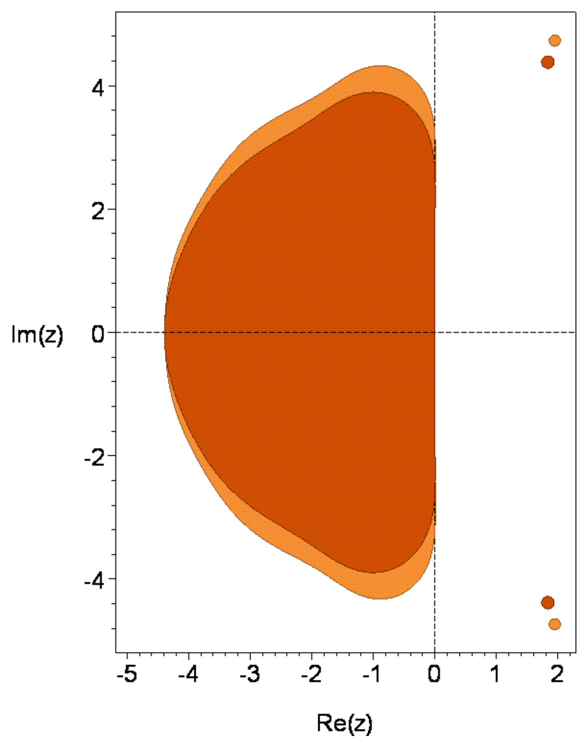
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2645443611 \times 10^{(-4)}$.

The principal error norm of the order 5 embedded scheme is: $0.5730538106 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 15.87801592.

The 2-norm of the linking coefficients is: 26.36487825.

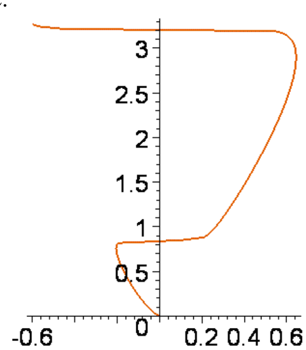
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively $[-4.3854, 0]$ and $[-4.3959, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [0.8482, 3.21385].

The coefficients are:
c[2]=7/95,
c[3]=35/243,
c[4]=35/162,
c[5]=31/55,
c[6]=97/140,
c[7]=35/36,
c[8]=1,
c[9]=1,

a[2,1]=7/95,
a[3,1]=385/118098,
a[3,2]=16625/118098,
a[4,1]=35/648,
a[4,2]=0,
a[4,3]=35/216,
a[5,1]=262364129/407618750,
a[5,2]=0,
a[5,3]=-996909687/407618750,
a[5,4]=482147154/203809375,
a[6,1]=-369745979892278000011/279274807605353000000,
a[6,2]=0,
a[6,3]=40382359908551133219/72070918091704000000,
a[6,4]=-5837069082360096304407/1395022708312545550000,
a[6,5]=13508218613909220883/22593673904425227520,
a[7,1]=756727539023617977739/325483930565422777440,
a[7,2]=0,
a[7,3]=-11344125743485787/1187841741251840,
a[7,4]=93403161519631272506393/11188675650069067742640,
a[7,5]=-3811592140111638097686625/3308811923451740272240896,
a[7,6]=102601928198000/102454959175119,
a[8,1]=250792505789354790081/66169955021507254750,
a[8,2]=0,
a[8,3]=-349400598312667239/22005305959929250,
a[8,4]=17728479258264420469702254/1289712860941995340762625,
a[8,5]=-61925787315598406758085/27764881218432626442221,
a[8,6]=43795320281930/26932915012729,
a[8,7]=-96493150422/1790131071575,
a[9,1]=1414477/22101450,
a[9,2]=0,
a[9,3]=0,
a[9,4]=6047117272638/18236245267975,
a[9,5]=31062711625/123650103496,
a[9,6]=27376502125/148102904499,
a[9,7]=26504253/76308925,
a[9,8]=-23459/131064,

b[1]=1414477/22101450,
b[2]=0,
b[3]=0,
b[4]=6047117272638/18236245267975,
b[5]=31062711625/123650103496,
b[6]=27376502125/148102904499,
b[7]=26504253/76308925,
b[8]=-23459/131064,

b*[1]=9659334433759/154907736963000,
b*[2]=0,
b*[3]=0,
b*[4]=21597935405438536443/63908374454260348250,
b*[5]=230190597481295825/1039987387676705088,
b*[6]=8367042728056775/37747068053062584,
b*[7]=28150317302067/97244486689000,
b*[8]=-77493123437/612413141440,
b*[9]=-1/160.

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