Verner's 1991 8 stage order, 6 Runge-Kutta scheme (b) with a 9 stage, order 5 FSAL embedded scheme

See: Some Ruge-Kutta Formula Pairs, by J.H.Verner,
SIAM Journal on Numerical Analysis, Vol. 28, No. 2 (April 1991), pages 496 to 511.

The nodes of the scheme are:

$$c_2 = \frac{1}{8},\ c_3 = \frac{1}{6},\ c_4 = \frac{1}{4},\ c_5 = \frac{1}{2},\ c_6 = \frac{3}{5},\ c_7 = \frac{4}{5},\ c_8 = 1,\ c_9 = 1.$$

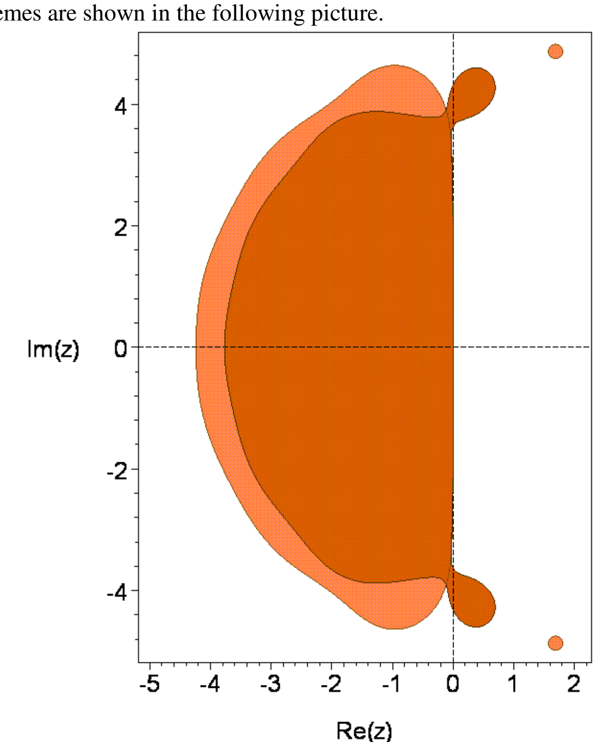
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1027690204 \times 10^{(-3)}$.

The principal error norm of the order 5 embedded scheme is: $0.5940092720 \times 10^{(-3)}$.

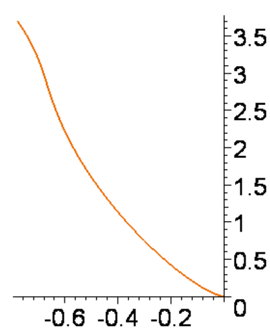
The maximum magnitude of the linking coefficients is: $\frac{74}{25} = 2.96$.

The 2-norm of the linking coefficients is: 4.753276999.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.23999, 0]$ and $[-3.7692, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis only at the origin.

The "Butcher" tableau of the scheme is as follows.

$$\begin{bmatrix}
\frac{1}{8} & \frac{1}{8} & & & & & & & & \\
\frac{1}{6} & \frac{1}{18} & \frac{1}{9} & & & & & & & \\
\frac{1}{4} & \frac{1}{16} & 0 & \frac{3}{16} & & & & & & \\
\frac{1}{2} & \frac{1}{4} & 0 & -\frac{3}{4} & 1 & & & & & \\
\frac{3}{5} & \frac{134}{625} & 0 & -\frac{333}{625} & \frac{476}{625} & \frac{98}{625} & & & & \\
\frac{4}{5} & -\frac{98}{1875} & 0 & \frac{12}{625} & \frac{10736}{13125} & -\frac{1936}{1875} & \frac{22}{21} & & & \\
1 & \frac{9}{50} & 0 & \frac{21}{25} & -\frac{2924}{1925} & \frac{74}{25} & -\frac{15}{7} & \frac{15}{22} & & \\
1 & \frac{11}{144} & 0 & 0 & \frac{256}{693} & 0 & \frac{125}{504} & \frac{125}{528} & \frac{5}{72} & \\
 & \frac{1}{18} & 0 & 0 & \frac{32}{63} & -\frac{2}{3} & \frac{125}{126} & 0 & -\frac{5}{63} & \frac{4}{21}
\end{bmatrix}$$

The last-but-one row gives the weights for the order 6 scheme.

The coefficients are:
c[2]=1/8,
c[3]=1/6,
c[4]=1/4,
c[5]=1/2,
c[6]=3/5,
c[7]=4/5,
c[8]=1,
c[9]=1,

a[2,1]=1/8,
a[3,1]=1/18,
a[3,2]=1/9,
a[4,1]=1/16,
a[4,2]=0,
a[4,3]=3/16,
a[5,1]=1/4,
a[5,2]=0,
a[5,3]=-3/4,
a[5,4]=1,
a[6,1]=134/625,
a[6,2]=0,
a[6,3]=-333/625,
a[6,4]=476/625,
a[6,5]=98/625,
a[7,1]=-98/1875,
a[7,2]=0,
a[7,3]=12/625,
a[7,4]=10736/13125,
a[7,5]=-1936/1875,
a[7,6]=22/21,
a[8,1]=9/50,
a[8,2]=0,
a[8,3]=21/25,

a[8,4]=-2924/1925,
a[8,5]=74/25,
a[8,6]=-15/7,
a[8,7]=15/22,
a[9,1]=11/144,
a[9,2]=0,
a[9,3]=0,
a[9,4]=256/693,
a[9,5]=0,
a[9,6]=125/504,
a[9,7]=125/528,
a[9,8]=5/72,

b[1]=11/144,
b[2]=0,
b[3]=0,
b[4]=256/693,
b[5]=0,
b[6]=125/504,
b[7]=125/528,
b[8]=5/72,

b*[1]=1/18,
b*[2]=0,
b*[3]=0,
b*[4]=32/63,
b*[5]=-2/3,
b*[6]=125/126,
b*[7]=0,
b*[8]=-5/63,
b*[9]=4/21.

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