Verner's 1991 8 stage order, 6 Runge-Kutta scheme (a) with a 9 stage, order 5 FSAL embedded scheme

See: Some Ruge-Kutta Formula Pairs, by J.H.Verner,
SIAM Journal on Numerical Analysis, Vol. 28, No. 2 (April 1991), pages 496 to 511.

The nodes of the scheme are:

$$c_2=\frac{1}{8},\; c_3=\frac{4}{9}-\frac{4\sqrt{10}}{45},\; c_4=\frac{2}{3}-\frac{2\sqrt{10}}{15},\; c_5=\frac{9}{16},\; c_6=\frac{1}{2},\; c_7=\frac{9}{10},\; c_8=1,\; c_9=1.$$

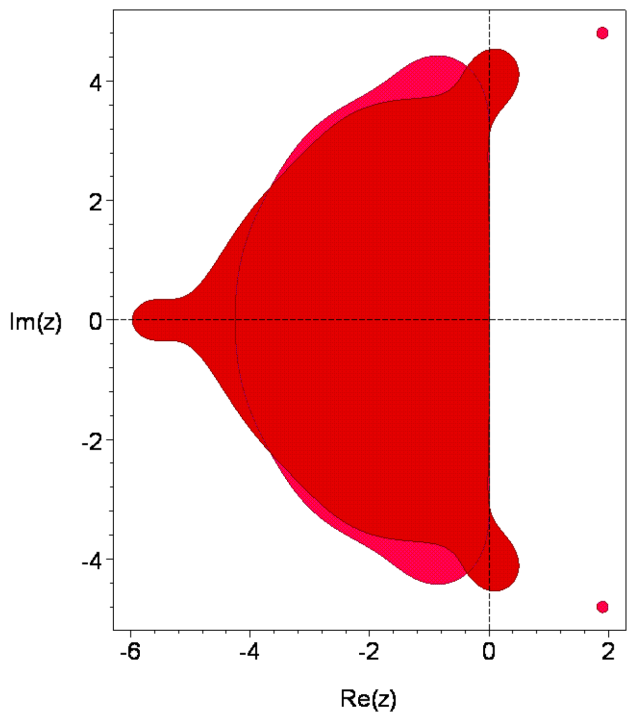
The principal error norm, that is, the 2-norm of the principal error terms is: $0.4931198171 \times 10^{(-4)}$.

The principal error norm of the order 5 embedded scheme is: $0.6365283308 \times 10^{(-3)}$.

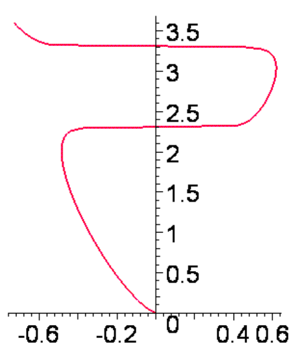
The maximum magnitude of the linking coefficients is: $\frac{17017}{1116}+\frac{5075\sqrt{10}}{1116} \simeq 29.62863721$.

The 2-norm of the linking coefficients is: 44.24632548.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.2506, 0]$ and $[-5.9700, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[2.3006, 3.3029]$.

The coefficients are:
c[2]=1/8,
c[3]=4/9-4/45*10^(1/2),
c[4]=2/3-2/15*10^(1/2),
c[5]=9/16,
c[6]=1/2,
c[7]=9/10,
c[8]=1,
c[9]=1,

a[2,1]=1/8,
a[3,1]=-268/405+92/405*10^(1/2),
a[3,2]=448/405-128/405*10^(1/2),
a[4,1]=1/6-1/30*10^(1/2),
a[4,2]=0,
a[4,3]=1/2-1/10*10^(1/2),
a[5,1]=11547/32768+405/16384*10^(1/2),
a[5,2]=0,
a[5,3]=-18225/32768-5103/16384*10^(1/2),
a[5,4]=12555/16384+2349/8192*10^(1/2),
a[6,1]=19662371/51149376+441281/12787344*10^(1/2),
a[6,2]=0,
a[6,3]=-3786045/5683264-252663/710408*10^(1/2),
a[6,4]=1570556745/1821486112+290041461/910743056*10^(1/2),
a[6,5]=-41227072/512292969+1374464/512292969*10^(1/2),
a[7,1]=-154207593/369412160-1829424339/11544130000*10^(1/2),
a[7,2]=0,
a[7,3]=2659895739/1847060800+653855409/1154413000*10^(1/2),
a[7,4]=-349492176711/591982986400-359784638379/1479957466000*10^(1/2),
a[7,5]=153920585664/92497341625+311066673408/462486708125*10^(1/2),
a[7,6]=-1944/1625-6804/8125*10^(1/2),
a[8,1]=70594945601/21406013856+21473424323/21406013856*10^(1/2),
a[8,2]=0,
a[8,3]=-794525145/88090592-249156075/88090592*10^(1/2),
a[8,4]=866290968775/254097312624+256998959765/254097312624*10^(1/2),
a[8,5]=-15964196472448/1286367645159-5039429245312/1286367645159*10^(1/2),
a[8,6]=17017/1116+5075/1116*10^(1/2),
a[8,7]=42875/90396+16625/90396*10^(1/2),
a[9,1]=31/324-37/4860*10^(1/2),
a[9,2]=0,
a[9,3]=0,
a[9,4]=37435/69228-3235/69228*10^(1/2),
a[9,5]=-1245184/1090341+9699328/16355115*10^(1/2),
a[9,6]=71/54-74/135*10^(1/2),
a[9,7]=625/486-250/729*10^(1/2),
a[9,8]=-23/21+37/105*10^(1/2),

b[1]=31/324-37/4860*10^(1/2),
b[2]=0,
b[3]=0,
b[4]=37435/69228-3235/69228*10^(1/2),
b[5]=-1245184/1090341+9699328/16355115*10^(1/2),
b[6]=71/54-74/135*10^(1/2),
b[7]=625/486-250/729*10^(1/2),
b[8]=-23/21+37/105*10^(1/2),

b*[1]=5/54-2/135*10^(1/2),
b*[2]=0,
b*[3]=0,
b*[4]=2390/17307+2290/17307*10^(1/2),
b*[5]=40960/121149+262144/605745*10^(1/2),
b*[6]=2/27-64/135*10^(1/2),
b*[7]=0,
b*[8]=150029/443709-236267/2218545*10^(1/2),
b*[9]=2411/126774+1921/63387*10^(1/2).

Version: 5 Nov 2011, Peter Stone