Verner's "most efficient" 8 stage, order 6 Runge-Kutta scheme with a 9 stage, order 5 FSAL embedded scheme

The coefficients are available from Jim Verner's website at: http://people.math.sfu.ca/~jverner/

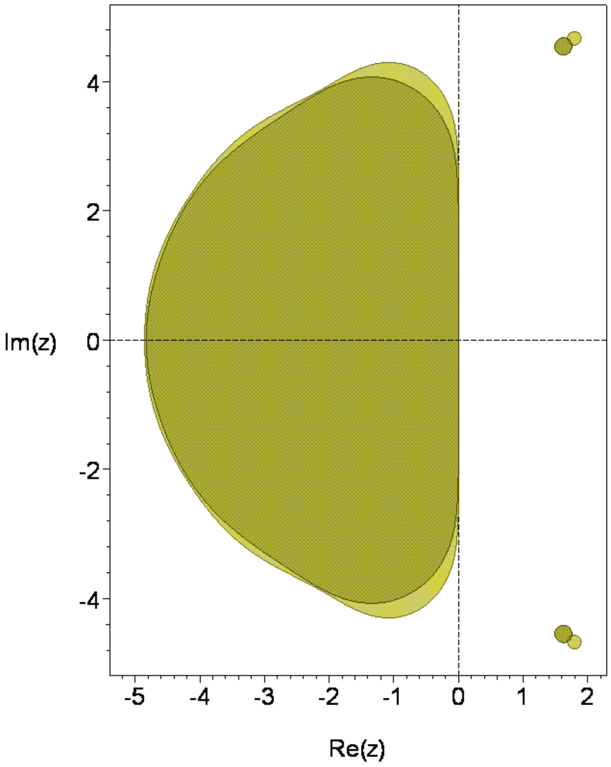
The nodes of the scheme are:

$$c_2 = \frac{3}{50},\ c_3 = \frac{1439}{15000},\ c_4 = \frac{1439}{10000},\ c_5 = \frac{4973}{10000},\ c_6 = \frac{389}{400},\ c_7 = \frac{1999}{2000},\ c_8 = 1,\ c_9 = 1.$$

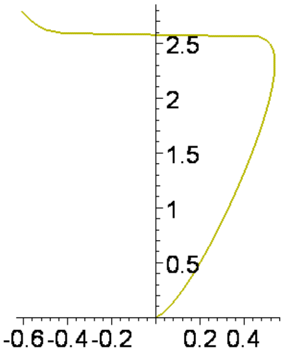
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1446174055 \text{ x } 10^{(-5)}$.

The principal error norm of the order 5 embedded scheme is: $0.1319717314 \text{ x } 10^{(-2)}$.
The maximum magnitude of the linking coefficients is: 207.9528063.
The 2-norm of the linking coefficients is: 495.7182555.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.
The real stability intervals of the order 6 and 5 schemes are respectively $[-4.8553, 0]$ and $[-4.8309, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 2.5842]$.

The coefficients are:
c[2]=3/50,
c[3]=1439/15000,
c[4]=1439/10000,
c[5]=4973/10000,
c[6]=389/400,
c[7]=1999/2000,
c[8]=1,
c[9]=1,

a[2,1]=3/50,
a[3,1]=519479/27000000,
a[3,2]=2070721/27000000,
a[4,1]=1439/40000,
a[4,2]=0,
a[4,3]=4317/40000,
a[5,1]=109225017611/82828840000,
a[5,2]=0,
a[5,3]=-417627820623/82828840000,
a[5,4]=43699198143/10353605000,
a[6,1]=-8036815292643907349452552172369/191934985946683241245914401600,
a[6,2]=0,
a[6,3]=246134619571490020064824665/1543816496655405117602368,
a[6,4]=-13880495956885686234074067279/113663489566254201783474344,
a[6,5]=755005057777788994734129/136485922925633667082436,
a[7,1]=-1663299841566102097180506666498880934230261/30558424506156170307020957791311384232000,
a[7,2]=0,
a[7,3]=130838124195285491799043628811093033/631862949514135618861563657970240,
a[7,4]=-3287100453856023634160618787153901962873/20724314915376755629135711026851409200,
a[7,5]=2771826790140332140865242520369241/396438716042723436917079980147600,
a[7,6]=-1799166916139193/96743806114007800,
a[8,1]=-832144750039369683895428386437986853923637763/15222974550069600748763651844667619945204887,
a[8,2]=0,
a[8,3]=818622075710363565982285196611368750/3936576237903728151856072395343129,
a[8,4]=-9818985165491658464841194581385463434793741875/61642597962658994069869370923196463581866011,
a[8,5]=31796692141848558720425711042548134769375/4530254033500045975557858016006308628092,
a[8,6]=-14064542118843830075/766928748264306853644,
a[8,7]=-1424670304836288125/2782839104764768088217,
a[9,1]=382735282417/11129397249634,
a[9,2]=0,
a[9,3]=0,
a[9,4]=5535620703125000/21434089949505429,
a[9,5]=13867056347656250/32943296570459319,
a[9,6]=626271188750/142160006043,
a[9,7]=-51160788125000/289890548217,
a[9,8]=163193540017/946795234,

b[1]=382735282417/11129397249634,
b[2]=0,
b[3]=0,
b[4]=5535620703125000/21434089949505429,
b[5]=13867056347656250/32943296570459319,
b[6]=626271188750/142160006043,
b[7]=-51160788125000/289890548217,
b[8]=163193540017/946795234,

b*[1]=1243106378698856756467986 13/2890072468789466426596827670,
b*[2]=0,
b*[3]=0,
b*[4]=26586315173716499036133092 1875/11131972714633723039403193 69579,
b*[5]=30754935571740308065363029 53125/68437499220423238765469496 99876,
b*[6]=6779800000873387981326305 5/295327921476667375500363 72,
b*[7]=-1099436585155390846238326 375/1505570649644640885919616 7,
b*[8]=2617125265308637318157180 2/3687944788907323460335 05,
b*[9]=1/30.

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