A modification of the Prince-Dormand 8 stage, combined order 5 and 6 Runge-Kutta scheme

See: P.J. Prince and J. R. Dormand, High order embedded Runge-Kutta formulae,
Journal of Computational and Applied Mathematics . 7 (1981), pp. 67-75.

The nodes of the scheme are:

$$c_2 = \frac{7}{39},\ c_3 = \frac{2}{9},\ c_4 = \frac{3}{7},\ c_5 = \frac{23}{33},\ c_6 = \frac{24}{31},\ c_7 = 1,\ c_8 = 1.$$

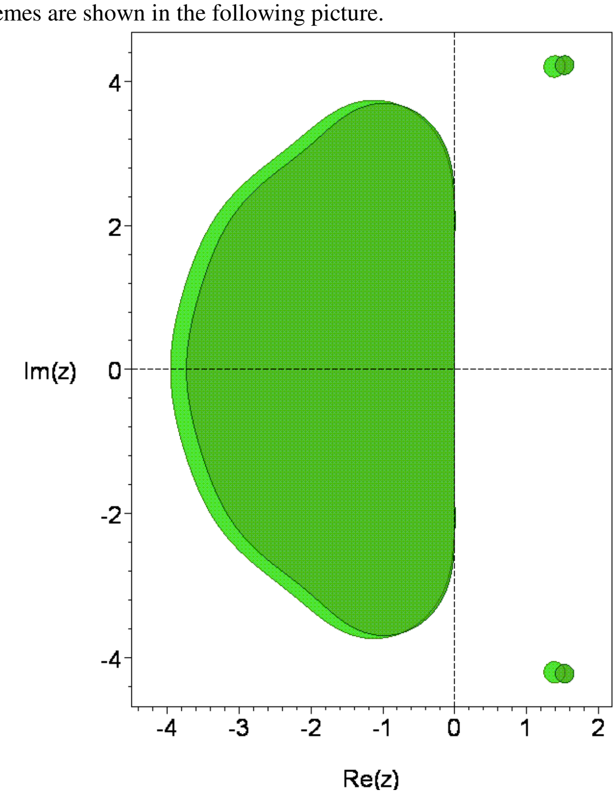
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2106308767 \times 10^{(-3)}$.

The principal error norm of the order 5 embedded scheme is: $0.1824880258 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: $\frac{1592286101}{1436292000} \simeq 1.108608905$.

The 2-norm of the linking coefficients is: 2.515167033.

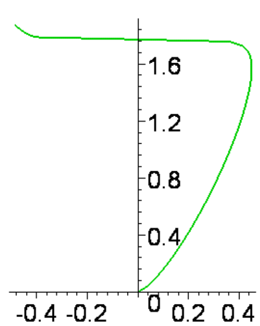
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively $[-3.9541, 0]$ and $[-3.7319, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 1.7644]$.

The "Butcher" tableau of the scheme is as follows.

$$
\left[\begin{array}{ccccccccc}
\frac{7}{39} & \frac{7}{39} & & & & & & & \\
\frac{2}{9} & \frac{16}{189} & \frac{26}{189} & & & & & & \\
\frac{3}{7} & \frac{957}{9604} & -\frac{1053}{2401} & \frac{1053}{1372} & & & & & \\
\frac{23}{33} & \frac{2563741}{11068596} & -\frac{18239}{102487} & \frac{3243}{761332} & \frac{3284078}{5138991} & & & & \\
\frac{24}{31} & -\frac{11597952}{148686881} & \frac{92664}{208537} & \frac{98740944}{564271331} & -\frac{26004300}{372178963} & \frac{9368775900}{30948112231} & & & \\
1 & \frac{38665819}{91808640} & -\frac{897}{1232} & \frac{156399}{1505504} & \frac{1592286101}{1436292000} & -\frac{2279466607}{2965053280} & \frac{972169703}{1126224000} & & \\
1 & \frac{118627013}{607606272} & -\frac{1527}{3136} & \frac{26560509}{49818496} & \frac{576719677}{1357948800} & -\frac{6116292391}{16604298368} & \frac{5233891417}{7453555200} & 0 & \\
b & \frac{14459}{198720} & 0 & \frac{19683}{68432} & \frac{8252237}{43524000} & \frac{143496441}{1058947600} & \frac{28629151}{119448000} & \frac{11}{1120} & \frac{13}{200} \\
b* & \frac{1236443}{16593120} & 0 & \frac{43680951}{157136980} & \frac{379485253}{1817127000} & \frac{1629060147}{17684424920} & \frac{30137260793}{109712988000} & \frac{12}{167} & 0
\end{array}\right]
$$

The last-but-one row gives the weights for the order 6 scheme while the last row gives the weights for the embedded order 5 scheme.

The coefficients are:
c[2]=7/39,
c[3]=2/9,
c[4]=3/7,
c[5]=23/33,
c[6]=24/31,
c[7]=1,
c[8]=1,

a[2,1]=7/39,
a[3,1]=16/189,
a[3,2]=26/189,
a[4,1]=957/9604,
a[4,2]=-1053/2401,
a[4,3]=1053/1372,
a[5,1]=2563741/11068596,
a[5,2]=-18239/102487,
a[5,3]=3243/761332,
a[5,4]=3284078/5138991,
a[6,1]=-11597952/148686881,
a[6,2]=92664/208537,
a[6,3]=98740944/564271331,
a[6,4]=-26004300/372178963,
a[6,5]=9368775900/30948112231,
a[7,1]=38665819/91808640,
a[7,2]=-897/1232,
a[7,3]=156399/1505504,
a[7,4]=1592286101/1436292000,
a[7,5]=-2279466607/2965053280,
a[7,6]=972169703/1126224000,
a[8,1]=118627013/607606272,
a[8,2]=-1527/3136,
a[8,3]=26560509/49818496,

a[8,4]=576719677/1357948800,
a[8,5]=-6116292391/16604298368,
a[8,6]=5233891417/7453555200,
a[8,7]=0,

b[1]=14459/198720,
b[2]=0,
b[3]=19683/68432,
b[4]=8252237/43524000,
b[5]=143496441/1058947600,
b[6]=28629151/119448000,
b[7]=11/1120,
b[8]=13/200,

b*[1]=1236443/16593120,
b*[2]=0,
b*[3]=43680951/157136980,
b*[4]=379485253/1817127000,
b*[5]=1629060147/17684424920,
b*[6]=30137260793/109712988000,
b*[7]=12/167,
b*[8]=0.

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