Prince-Dormand 8 stage, combined order 5 and 6 Runge-Kutta scheme

See: P.J. Prince and J. R. Dormand, High order embedded Runge-Kutta formulae,
Journal of Computational and Applied Mathematics . 7 (1981), pp. 67-75.

The nodes of the scheme are:

$$c_2 = \frac{1}{10},\ c_3 = \frac{2}{9},\ c_4 = \frac{3}{7},\ c_5 = \frac{3}{5},\ c_6 = \frac{4}{5},\ c_7 = 1,\ c_8 = 1.$$

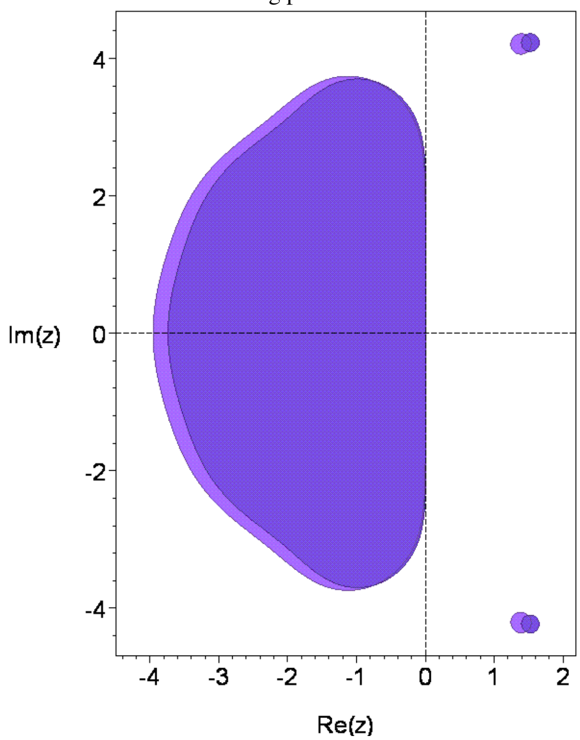
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2326287006 \times 10^{(-3)}$.

The principal error norm of the order 5 embedded scheme is: $0.1845470108 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: $\frac{899983}{200772} \simeq 4.482612117$.

The 2-norm of the linking coefficients is: 8.745463403.

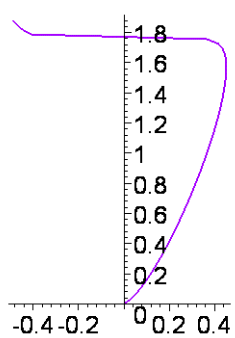
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 5 scheme appears in the darker shade.

The real stability intervals of the order 6 and 5 schemes are respectively $[-3.9541, 0]$ and $[-3.7373, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 6 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 1.7644]$.

The "Butcher" tableau of the scheme is as follows.

$$\begin{bmatrix}
\frac{1}{10} & \frac{1}{10} & & & & & & & \\
\frac{2}{9} & -\frac{2}{81} & \frac{20}{81} & & & & & & \\
\frac{3}{7} & \frac{615}{1372} & -\frac{270}{343} & \frac{1053}{1372} & & & & & \\
\frac{3}{5} & \frac{3243}{5500} & -\frac{54}{55} & \frac{50949}{71500} & \frac{4998}{17875} & & & & \\
\frac{4}{5} & -\frac{26492}{37125} & \frac{72}{55} & \frac{2808}{23375} & -\frac{24206}{37125} & \frac{338}{459} & & & \\
1 & \frac{5561}{2376} & -\frac{35}{11} & -\frac{24117}{31603} & \frac{899983}{200772} & -\frac{5225}{1836} & \frac{3925}{4056} & & \\
1 & \frac{465467}{266112} & -\frac{2945}{1232} & -\frac{5610201}{14158144} & \frac{10513573}{3212352} & -\frac{424325}{205632} & \frac{376225}{454272} & 0 & \\
b & \frac{61}{864} & 0 & \frac{98415}{321776} & \frac{16807}{146016} & \frac{1375}{7344} & \frac{1375}{5408} & -\frac{37}{1120} & \frac{1}{10} \\
b^* & \frac{821}{10800} & 0 & \frac{19683}{71825} & \frac{175273}{912600} & \frac{395}{3672} & \frac{785}{2704} & \frac{3}{50} & 0
\end{bmatrix}$$

The last-but-one row gives the weights for the order 6 scheme while the last row gives the weights for the embedded order 5 scheme.

The coefficients are:
c[2]=1/10,
c[3]=2/9,
c[4]=3/7,
c[5]=3/5,
c[6]=4/5,
c[7]=1,
c[8]=1,

a[2,1]=1/10,
a[3,1]=-2/81,
a[3,2]=20/81,
a[4,1]=615/1372,
a[4,2]=-270/343,
a[4,3]=1053/1372,
a[5,1]=3243/5500,
a[5,2]=-54/55,
a[5,3]=50949/71500,
a[5,4]=4998/17875,
a[6,1]=-26492/37125,
a[6,2]=72/55,
a[6,3]=2808/23375,
a[6,4]=-24206/37125,
a[6,5]=338/459,
a[7,1]=5561/2376,
a[7,2]=-35/11,
a[7,3]=-24117/31603,
a[7,4]=899983/200772,
a[7,5]=-5225/1836,
a[7,6]=3925/4056,
a[8,1]=465467/266112,
a[8,2]=-2945/1232,
a[8,3]=-5610201/14158144,

a[8,4]=10513573/3212352,
a[8,5]=-424325/205632,
a[8,6]=376225/454272,
a[8,7]=0,

b[1]=61/864,
b[2]=0,
b[3]=98415/321776,
b[4]=16807/146016,
b[5]=1375/7344,
b[6]=1375/5408,
b[7]=-37/1120,
b[8]=1/10,

b*[1]=821/10800,
b*[2]=0,
b*[3]=19683/71825,
b*[4]=175273/912600,
b*[5]=395/3672,
b*[6]=785/2704,
b*[7]=3/50,
b*[8]=0.

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