

**A 7 stage, order 6 Runge-Kutta scheme with "simple" nodes**

For early schemes constructed by hand it was convenient to work with "simple" nodes.

For example, two early schemes of John Butcher have nodes which are rational numbers with numerator and denominator each a digit between 1 and 6.

The scheme considered here has nodes which are "simple" in this sense.

The nodes of the scheme are:

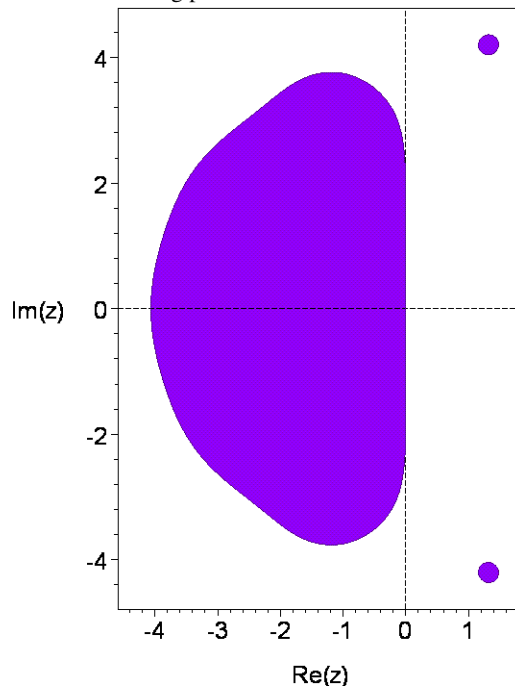
$$c_2 = \frac{1}{6}, c_3 = \frac{1}{5}, c_4 = \frac{1}{3}, c_5 = \frac{2}{3}, c_6 = \frac{3}{4}, c_7 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is:  $0.2484943086 \times 10^{(-3)}$ .

The maximum magnitude of the linking coefficients is:  $\frac{7}{6} \approx 1.166666667$ .

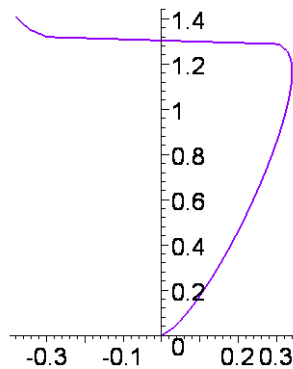
The 2-norm of the linking coefficients is: 2.159196208.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is  $[-4.0648, 0]$ .

The following picture shows the result of distorting the boundary curve of the stability region horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval:  $[0, 1.3068]$ .

The "Butcher" tableau of the scheme is as follows.

$$\left[ \begin{array}{ccccccc} \frac{1}{6} & \frac{1}{6} & & & & & \\ \frac{1}{5} & \frac{2}{25} & \frac{3}{25} & & & & \\ \frac{1}{3} & \frac{2}{27} & -\frac{1}{9} & \frac{10}{27} & & & \\ \frac{2}{3} & \frac{10}{27} & -\frac{2}{9} & -\frac{35}{54} & \frac{7}{6} & & \\ \frac{3}{4} & -\frac{9}{256} & \frac{9}{64} & \frac{165}{448} & 0 & \frac{495}{1792} & \\ 1 & \frac{4}{19} & -\frac{3}{19} & -\frac{305}{1463} & \frac{81}{95} & -\frac{90}{133} & \frac{1024}{1045} \\ & \frac{3}{40} & 0 & \frac{625}{3696} & \frac{27}{100} & \frac{27}{280} & \frac{256}{825} & \frac{19}{240} \end{array} \right]$$

The coefficients are as follows.

c[2]=1/6,  
c[3]=1/5,  
c[4]=1/3,  
c[5]=2/3,  
c[6]=3/4,  
c[7]=1,

a[2,1]=1/6,  
a[3,1]=2/25,  
a[3,2]=3/25,  
a[4,1]=2/27,  
a[4,2]=-1/9,  
a[4,3]=10/27,  
a[5,1]=10/27,  
a[5,2]=-2/9,  
a[5,3]=-35/54,  
a[5,4]=7/6,  
a[6,1]=-9/256,  
a[6,2]=9/64,  
a[6,3]=165/448,  
a[6,4]=0,  
a[6,5]=495/1792,  
a[7,1]=4/19,  
a[7,2]=-3/19,  
a[7,3]=-305/1463,  
a[7,4]=81/95,  
a[7,5]=-90/133,  
a[7,6]=1024/1045,

b[1]=3/40,  
b[2]=0,  
b[3]=625/3696,  
b[4]=27/100,  
b[5]=27/280,  
b[6]=256/825,  
b[7]=19/240.