John Butcher's stage-order 2, 7 stage, order 6 Runge-Kutta scheme A

See: On Runge-Kutta Processes of High Order, by J. C. Butcher,
Journal of the Australian Mathematical Society, Vol. 4, (1964), pages 179 to 194.

The nodes of the scheme are:

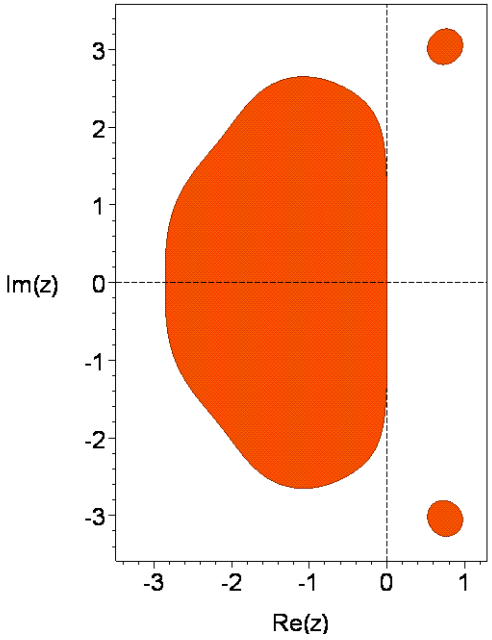
$$c_2 = \frac{1}{2},\ c_3 = \frac{2}{3},\ c_4 = \frac{1}{3},\ c_5 = \frac{5}{6},\ c_6 = \frac{1}{6},\ c_7 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.4944017072 \times 10^{(-2)}$.

The maximum magnitude of the linking coefficients is: $\frac{118}{39} \simeq 3.025641026$.

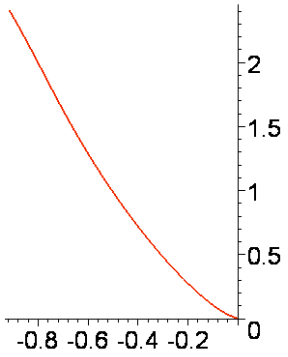
The 2-norm of the linking coefficients is: 4.873856558.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-2.8561, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region horizontally by taking the 11th root of the real part of points along the curve.



The stability region only intersects the nonnegative imaginary axis at the origin.

The "Butcher" tableau of the scheme is as follows.

$$\begin{bmatrix}
\frac{1}{2} & \frac{1}{2} & & & & & & \\
\frac{2}{3} & \frac{2}{9} & \frac{4}{9} & & & & & \\
\frac{1}{3} & \frac{7}{36} & \frac{2}{9} & -\frac{1}{12} & & & & \\
\frac{5}{6} & -\frac{35}{144} & -\frac{55}{36} & \frac{35}{48} & \frac{15}{8} & & & \\
\frac{1}{6} & -\frac{1}{360} & -\frac{11}{36} & -\frac{1}{8} & \frac{1}{2} & \frac{1}{10} & & \\
1 & -\frac{41}{260} & \frac{22}{13} & \frac{43}{156} & -\frac{118}{39} & \frac{32}{195} & \frac{80}{39} & \\
 & \frac{13}{200} & 0 & \frac{11}{40} & \frac{11}{40} & \frac{4}{25} & \frac{4}{25} & \frac{13}{200}
\end{bmatrix}$$

The coefficients are as follows.
c[2]=1/2,
c[3]=2/3,
c[4]=1/3,
c[5]=5/6,
c[6]=1/6,
c[7]=1,

a[2,1]=1/2,
a[3,1]=2/9,
a[3,2]=4/9,
a[4,1]=7/36,
a[4,2]=2/9,
a[4,3]=-1/12,
a[5,1]=-35/144,
a[5,2]=-55/36,
a[5,3]=35/48,
a[5,4]=15/8,
a[6,1]=-1/360,
a[6,2]=-11/36,
a[6,3]=-1/8,
a[6,4]=1/2,
a[6,5]=1/10,
a[7,1]=-41/260,
a[7,2]=22/13,
a[7,3]=43/156,
a[7,4]=-118/39,
a[7,5]=32/195,
a[7,6]=80/39,

b[1]=13/200,
b[2]=0,
b[3]=11/40,
b[4]=11/40,
b[5]=4/25,
b[6]=4/25,
b[7]=13/200.

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