A companion to Huta's 8 stage, order 6 Runge-Kutta scheme B

This scheme has the same nodes and weights as the Huta scheme. It also has the same coefficient for z^8 in the stability polynomial. However we take an alternative value for $a_{7,6}$ determined by specifying that $a_{6,5} = 8$.

The nodes of the scheme are:

$$c_2 = \frac{1}{9},\ c_3 = \frac{1}{6},\ c_4 = \frac{1}{3},\ c_5 = \frac{1}{2},\ c_6 = \frac{2}{3},\ c_7 = \frac{5}{6},\ c_8 = 1.$$

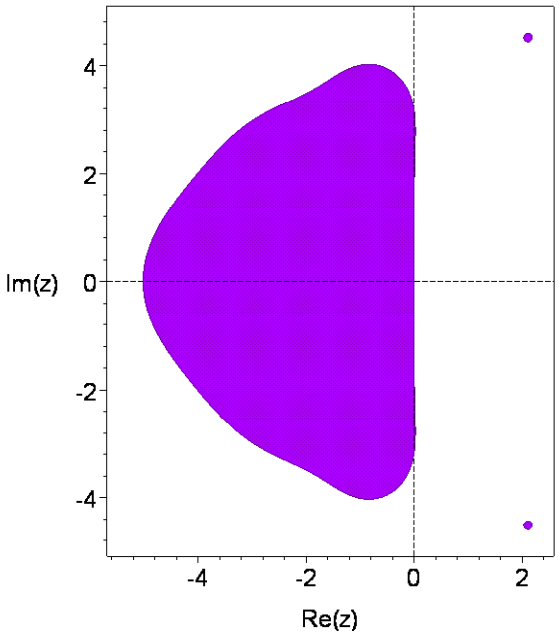
Note: The scheme satisfies the order 7 quadrature conditions.

The principal error norm, that is, the 2-norm of the principal error terms is: $0.5359206045 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: $\frac{8287}{317} \simeq 26.14195584$.

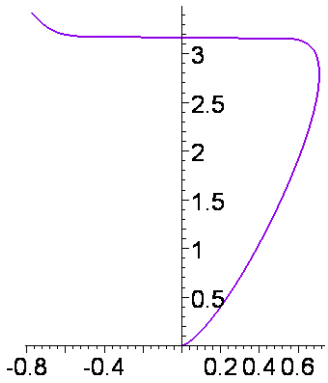
The 2-norm of the linking coefficients is: 37.10448027.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-5.0209, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 3.1695]$.

The Butcher tableau of the scheme is as follows.

$$\begin{bmatrix}
\frac{1}{9} & \frac{1}{9} & & & & & & & \\
\frac{1}{6} & \frac{1}{24} & \frac{1}{8} & & & & & & \\
\frac{1}{3} & \frac{1}{6} & -\frac{1}{2} & \frac{2}{3} & & & & & \\
\frac{1}{2} & \frac{935}{2536} & -\frac{2781}{2536} & \frac{309}{317} & \frac{321}{1268} & & & & \\
\frac{2}{3} & -\frac{12710}{951} & \frac{8287}{317} & -\frac{40}{317} & -\frac{6335}{317} & 8 & & & \\
\frac{5}{6} & \frac{5840285}{3104064} & -\frac{7019}{2536} & -\frac{52213}{86224} & \frac{1278709}{517344} & -\frac{433}{2448} & \frac{33}{1088} & & \\
1 & -\frac{5101675}{1767592} & \frac{112077}{25994} & \frac{334875}{441898} & -\frac{973617}{883796} & -\frac{1421}{1394} & \frac{333}{5576} & \frac{36}{41} & \\
 & \frac{41}{840} & 0 & \frac{9}{35} & \frac{9}{280} & \frac{34}{105} & \frac{9}{280} & \frac{9}{35} & \frac{41}{840}
\end{bmatrix}$$

The coefficients are:
c[2]=1/9,
c[3]=1/6,
c[4]=1/3,
c[5]=1/2,
c[6]=2/3,
c[7]=5/6,
c[8]=1,

a[2,1]=1/9,
a[3,1]=1/24,
a[3,2]=1/8,
a[4,1]=1/6,
a[4,2]=-1/2,
a[4,3]=2/3,
a[5,1]=935/2536,
a[5,2]=-2781/2536,
a[5,3]=309/317,
a[5,4]=321/1268,
a[6,1]=-12710/951,
a[6,2]=8287/317,
a[6,3]=-40/317,
a[6,4]=-6335/317,
a[6,5]=8,
a[7,1]=5840285/3104064,
a[7,2]=-7019/2536,
a[7,3]=-52213/86224,
a[7,4]=1278709/517344,
a[7,5]=-433/2448,
a[7,6]=33/1088,
a[8,1]=-5101675/1767592,
a[8,2]=112077/25994,
a[8,3]=334875/441898,
a[8,4]=-973617/883796,
a[8,5]=-1421/1394,
a[8,6]=333/5576,
a[8,7]=36/41,

b[1]=41/840,
b[2]=0,
b[3]=9/35,
b[4]=9/280,
b[5]=34/105,
b[6]=9/280,
b[7]=9/35,
b[8]=41/840.

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