Anton Huta's 8 stage, order 6 Runge-Kutta scheme B

See:

1. Une amélioration de la méthode de Runge-Kutta-Nyström pour la résolution numérique des équations différentielles du premièr ordre, by Anton Huta,
   Acta Fac. Nat. Univ. Comenian Math., Vol. 1, pages 201-224 (1956).

2. Contribution à la formule de sixième ordre dans la méthode de Runge-Kutta-Nyström, by Anton Huta,
   Acta Fac.Nat. Univ. Comenian Math., Vol. 2, pages 21-24 (1957).

The nodes of the scheme are:

$$c_2 = \frac{1}{9},\ c_3 = \frac{1}{6},\ c_4 = \frac{1}{3},\ c_5 = \frac{1}{2},\ c_6 = \frac{2}{3},\ c_7 = \frac{5}{6},\ c_8 = 1.$$

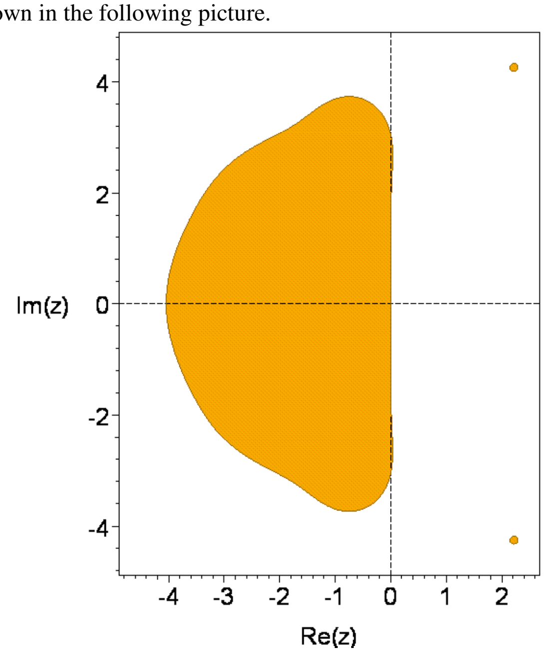
Note: The scheme satisfies the order 7 quadrature conditions.

The principal error norm, that is, the 2-norm of the principal error terms is: $0.1511955200 \times 10^{(-2)}$.

The maximum magnitude of the linking coefficients is: $\frac{91}{2} = 45.5$.

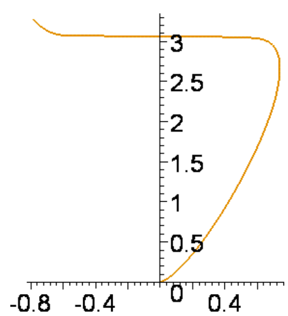
The 2-norm of the linking coefficients is: 56.65735528.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-4.0429, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[0, 3.0563]$.

The Butcher tableau of the scheme is as follows.

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & & & & & & & \\ \frac{1}{6} & \frac{1}{24} & \frac{1}{8} & & & & & & \\ \frac{1}{3} & \frac{1}{6} & -\frac{1}{2} & \frac{2}{3} & & & & & \\ \frac{1}{2} & \frac{139}{272} & -\frac{945}{544} & \frac{105}{68} & \frac{99}{544} & & & & \\ \frac{2}{3} & -\frac{53}{3} & \frac{91}{2} & -\frac{52}{3} & -\frac{107}{6} & 8 & & & \\ \frac{5}{6} & \frac{55487}{22824} & -\frac{83}{16} & \frac{2849}{1902} & \frac{34601}{15216} & -\frac{640}{2853} & \frac{107}{2536} & & \\ 1 & -\frac{101195}{25994} & \frac{351}{41} & -\frac{35994}{12997} & -\frac{26109}{25994} & -\frac{10000}{12997} & -\frac{36}{12997} & \frac{36}{41} & \\ & \frac{41}{840} & 0 & \frac{9}{35} & \frac{9}{280} & \frac{34}{105} & \frac{9}{280} & \frac{9}{35} & \frac{41}{840} \end{bmatrix}$$

The coefficients are:
c[2]=1/9,
c[3]=1/6,
c[4]=1/3,
c[5]=1/2,
c[6]=2/3,
c[7]=5/6,
c[8]=1,

a[2,1]=1/9,
a[3,1]=1/24,
a[3,2]=1/8,
a[4,1]=1/6,
a[4,2]=-1/2,
a[4,3]=2/3,
a[5,1]=139/272,
a[5,2]=-945/544,
a[5,3]=105/68,
a[5,4]=99/544,
a[6,1]=-53/3,
a[6,2]=91/2,
a[6,3]=-52/3,
a[6,4]=-107/6,
a[6,5]=8,
a[7,1]=55487/22824,
a[7,2]=-83/16,
a[7,3]=2849/1902,
a[7,4]=34601/15216,
a[7,5]=-640/2853,
a[7,6]=107/2536,
a[8,1]=-101195/25994,
a[8,2]=351/41,
a[8,3]=-35994/12997,
a[8,4]=-26109/25994,
a[8,5]=-10000/12997,
a[8,6]=-36/12997,
a[8,7]=36/41,

b[1]=41/840,
b[2]=0,
b[3]=9/35,
b[4]=9/280,
b[5]=34/105,
b[6]=9/280,
b[7]=9/35,
b[8]=41/840.

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