A 7 stage, combined order 4 and 5 Runge-Kutta scheme

The scheme considered here is constructed using an algorithm of Sharp and Smart.
See: Explicit Runge-Kutta Pairs with One More Derivative Evaluation than the Minimum, by P.W.Sharp and E.Smart, Siam Journal of Scientific Computing, Vol. 14, No. 2, pages. 338-348, March 1993.

Note: The scheme is constructed so as to have a stability function close to that of a scheme of Bogacki and Shampine.
The nodes of the scheme are:

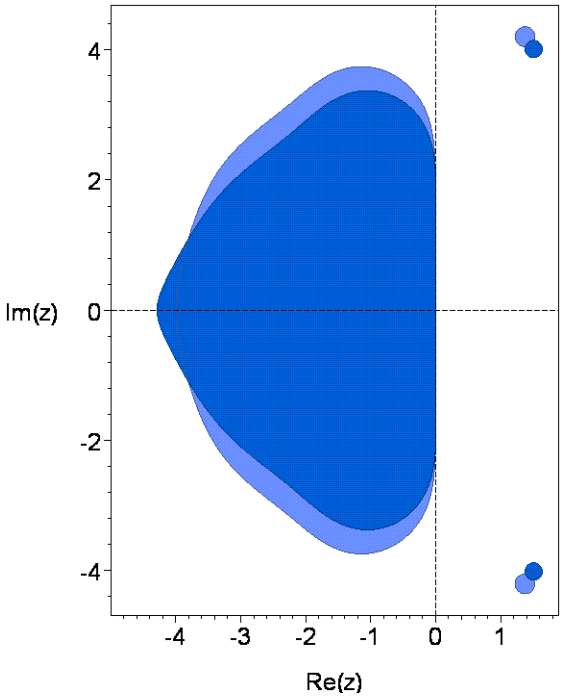
$$c_2 = \frac{143}{909},\ c_3 = \frac{143}{606},\ c_4 = \frac{307}{607},\ c_5 = \frac{93}{125},\ c_6 = \frac{777}{841},\ c_7 = 1.$$

The principal error norm of the order 5 scheme, that is, the 2-norm of the principal error terms is: $0.6376460570 \times 10^{(-4)}$.

The 2-norm of the order 7 error terms is $0.1603777822 \times 10^{(-3)}$, which is about 2.515 times the previous norm.

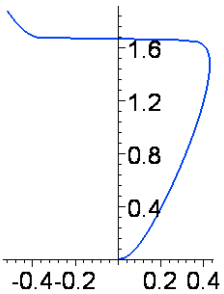
The principal error norm of the order 4 embedded scheme is: $0.8125271409 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 6.456166223.
The 2-norm of the linking coefficients is: 10.46301237.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.9879, 0]$ and $[-4.2885, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[0, 1.6639]$.

The coefficients are as follows:
c[2]=143/909,
c[3]=143/606,
c[4]=307/607,
c[5]=93/125,
c[6]=777/841,
c[7]=1,

a[2,1]=143/909,
a[3,1]=143/2424,
a[3,2]=143/808,
a[4,1]=2857824275327/9146778111614,
a[4,2]=-9567972715221/9146778111614,
a[4,3]=5668139195454/4573389055807,
a[5,1]=254424104877941/8957268246093750,
a[5,2]=158/249,
a[5,3]=-2407696795731566/4778105255859375,
a[5,4]=472715594543564809 31/8081165981914453 1250,
a[6,1]=-47806802807520996909341633167479808515193800578540115896/
264374847091816749657470764351518416386670778340337383 05,
a[6,2]=-3258579035192863483944739343435639/523243426969629227976204925685994 0,
a[6,3]=18209791143335404962195617869023901860171784423990752757 6/
282052699930542415720719580793681417751148608521445934 05,
a[6,4]=-87389011752504383198523171658444632280568663139100660253947/
159096591560938641035839158858678705849580407082 3275678843 80,
a[6,5]=17058334744266520885884345442500000/713253533176999565206944502046623 7,
a[7,1]=34184988557995143579838700512613706626776305230 35/
221675861649960554748670846695948376848701963673 6,
a[7,2]=-16053818347898736758233/27321881256640248385536,
a[7,3]=-71747748040485875058414151570341465185661044564255/
258177122943370245353852178685204700545461500451 84,
a[7,4]=94824231812339740024513302786842687546349368687256 7/
260770830038854456690383952607340691019496991838208,
a[7,5]=-82005183937188577798592626386718 75/92704585608790603547093805041989 92,
a[7,6]=47654005665537155/654961074633682992,

b[1]=210204947377/2849240535372,
b[2]=0,
b[3]=480613058832712388 4/14330999159156446490 91,
b[4]=479990038268371236 7/26912619953042079424,
b[5]=434193205336914062 5/14285181000773104128,
b[6]=400770187647167473 55/755291408140531616256,
b[7]=54/973,

b*[1]=817455315005692894881588654571039/111613552979073775132192148639980 80,
b*[2]=0,
b*[3]=105115859708264144701847397975049645110 1/311882900716489666402249910611121777768 0,
b*[4]=215039085847558894665938369071459741722 13/121644283354847329649180791388046083020800,
b*[5]=260602093856214253319512235079062890625/860914871147356612953111069735105773568,
b*[6]=453171196733118474387827067988552622083/784803647923581260434221894056151048192 0,
b*[7]=21/400.

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