A 7 stage, combined order 4 and 5 Runge-Kutta scheme

The scheme considered here is constructed using an algorithm of Sharp and Smart.
See: Explicit Runge-Kutta Pairs with One More Derivative Evaluation than the Minimum, by P.W.Sharp and E.Smart, Siam Journal of Scientific Computing, Vol. 14, No. 2, pages. 338-348, March 1993.

The nodes of the scheme are:

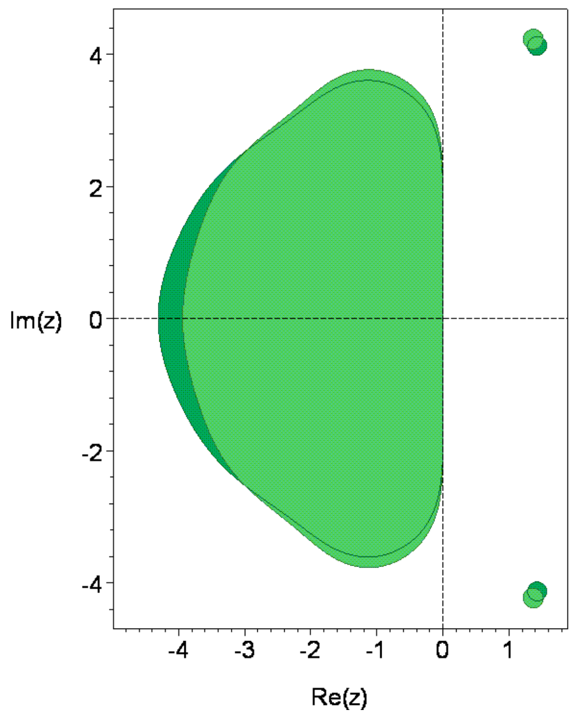
$$c_2 = \frac{100}{663},\ c_3 = \frac{50}{221},\ c_4 = \frac{398}{913},\ c_5 = \frac{269}{407},\ c_6 = \frac{158}{203},\ c_7 = 1.$$

The principal error norm of the order 5 scheme, that is, the 2-norm of the principal error terms is: $0.4451480595 \times 10^{(-4)}$.

The 2-norm of the order 7 error terms is $0.1727640516 \times 10^{(-3)}$, which is about 3.88105 times the previous norm.

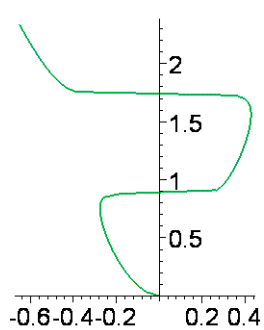
The principal error norm of the order 4 embedded scheme is: $0.6628561818 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 0.9896170728.
The 2-norm of the linking coefficients is: 2.223845466.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.9409, 0]$ and $[-4.3099, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0.88015, 1.7364].

The coefficients are as follows:
c[2]=100/663,
c[3]=50/221,
c[4]=398/913,
c[5]=269/407,
c[6]=158/203,
c[7]=1,

a[2,1]=100/663,
a[3,1]=25/442,
a[3,2]=75/442,
a[4,1]=185691677393/951310621250,
a[4,2]=-511535185629/951310621250,
a[4,3]=370272042868/475655310625,
a[5,1]=2266749954401441/8652486393536700,
a[5,2]=-45/107,
a[5,3]=748280457304967/2080932913082100,
a[5,4]=76529927789440303/166395179244432138,
a[6,1]=-24784906174562375576562812546108062005031/202914270414369776214747536626486030178750,
a[6,2]=4091393174924823520837970854911/8160443100775955740300789951250,
a[6,3]=55232760277489986286920046490105216694871/439210035324930128179596222153416054726250,
a[6,4]=-6871002337396701224170421575262095425649/85139473910986675954646665582744174857546,
a[6,5]=188/531,
a[7,1]=2059885076436524375226324554286533481429967/5473156158081776229718684667977063008000000,
a[7,2]=-2772217191888252407801/4375403181389245500000,
a[7,3]=6030294086928106094195125548744535863439681/23693406132757840088933843407023377472000000,
a[7,4]=4545204056746029627509276911576123354470587/4592891717080141264005384018501294183321600,
a[7,5]=-20754190161796749718605153161/25276817567759416891456550400,
a[7,6]=1456994449663963/1746628843837440,

b[1]=896852459/12155572000,
b[2]=0,
b[3]=400714760366709919/1352050846933248000,
b[4]=8972327793297/50775049270400,
b[5]=2995186678192093/27490810472064810,
b[6]=295769873281784489/1087699026783283200,
b[7]=31/429,

b*[1]=3215412654264102744764570088139/37784840235587726219406175291584,
b*[2]=0,
b*[3]=289446221570202899073167983054303/1179790366718086205469324401823744,
b*[4]=1029168095857103319368017191095142047/3945777139562332283276867742341118720,
b*[5]=195115505842420280611628854131673/7164071619405296801323096171064872,
b*[6]=33171284415115690721605525431890669/107017983499076800719948392027688960,
b*[7]=45/629.

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