A 7 stage, combined order 4 and 5 Runge-Kutta scheme

The scheme considered here is constructed using an algorithm of Sharp and Smart.
See: Explicit Runge-Kutta Pairs with One More Derivative Evaluation than the Minimum, by P.W.Sharp and E.Smart,
Siam Journal of Scientific Computing, Vol. 14, No. 2, pages. 338-348, March 1993.

The nodes of the scheme are:

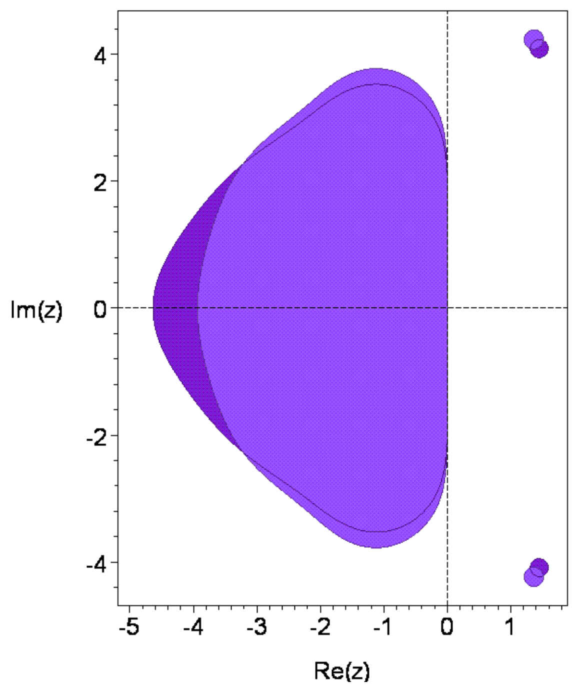
$$c_2 = \frac{88}{573},\ c_3 = \frac{44}{191},\ c_4 = \frac{364}{801},\ c_5 = \frac{465}{647},\ c_6 = \frac{148}{187},\ c_7 = 1.$$

The principal error norm of the order 5 scheme, that is, the 2-norm of the principal error terms is: $0.6137120017 \times 10^{(-4)}$.

The 2-norm of the order 7 error terms is $0.1541489682 \times 10^{(-3)}$, which is about 2.512 times the previous norm.

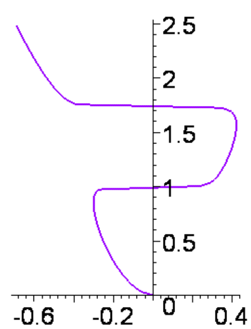
The principal error norm of the order 4 embedded scheme is: $0.6628561818 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 0.8719147941.
The 2-norm of the linking coefficients is: 1.948031719.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.9351, 0]$ and $[-4.6343, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[1.0002, 1.7347]$.

The coefficients are as follows:
c[2]=88/573,
c[3]=44/191,
c[4]=364/801,
c[5]=465/647,
c[6]=148/187,
c[7]=1,

a[2,1]=88/573,
a[3,1]=11/191,
a[3,2]=33/191,
a[4,1]=13560205841/62184610521,
a[4,2]=-13173737759/20728203507,
a[4,3]=54219681880/62184610521,
a[5,1]=116276184503811611/559629879688797456,
a[5,2]=47/2927,
a[5,3]=-494928406778711309/3587156371677632160,
a[5,4]=9197376305273583767 1/145334183907060430240,
a[6,1]=-14852890525615072155035186603296580850 95/61430951581015558274494226674199126468 64,
a[6,2]=15281460776743871983338124837/25407593197579730298365309919,
a[6,3]=792358320217542005224743723743959521617/262509725778760076994472960657136132553 6,
a[6,4]=-2353935986836386905097375854856628853 97/10743051832451858675898694471949525840 96,
a[6,5]=311/891,
a[7,1]=1041891499056963495191333464143855794311691/314979456432619250092539328897229820958 0800,
a[7,2]=-15183590541196607566 37/2282622152471533237365,
a[7,3]=217351394156486594328903468593029897372 87/363779867745799923579160078229813372160 0,
a[7,4]=721464195811123673260439864943817299767 31/183612144449719210508659799928528351910 400,
a[7,5]=-7225981367469745318852 49/3834550141687417097041200,
a[7,6]=24173098995529/45400921735600,

b[1]=1545613637/20666646000,
b[2]=0,
b[3]=10458135898012399/34201737080280000,
b[4]=100171496741812407/539170093418320000,
b[5]=363303947233051000 3/2029439662215884100 0,
b[6]=4520369512163923/24706548107280000,
b[7]=43/600,

b*[1]=21253024069483454243477028518 9/2691867881196436739241927081600,
b*[2]=0,
b*[3]=426663230040443615837686365938497897/148494596119646003831713365333369600 0,
b*[4]=152474114558251628999240672211481122 93/702278762105112711577766412046334720 00,
b*[5]=207935855870339110227987810622986363 31/165211358060503752233868714756894047 100,
b*[6]=236891787201388457695546250308185479/107269080342018483989125744674009600 0,
b*[7]=23/329.

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