Sharp and Smart's 7 stage, combined order 4 and 5 Runge-Kutta scheme

See: Explicit Runge-Kutta Pairs with One More Derivative Evaluation than the Minimum, by P.W.Sharp and E.Smart,
Siam Journal of Scientific Computing, Vol. 14, No. 2, pages. 338-348, March 1993.

The nodes of the scheme are:

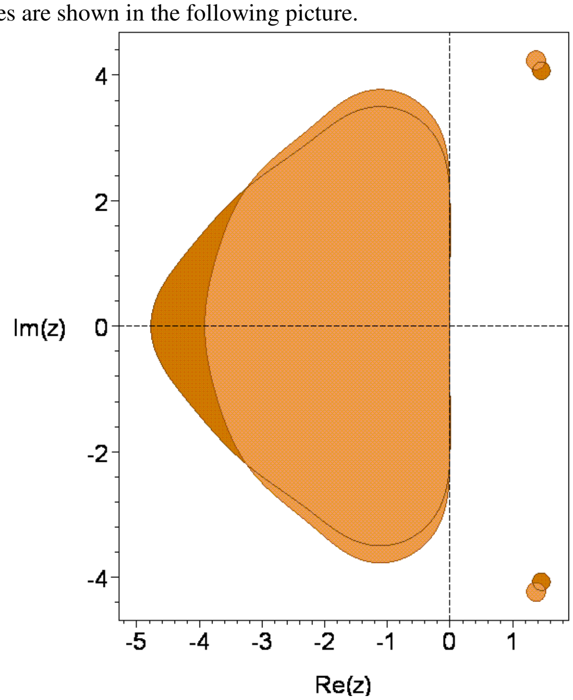
$$c_2 = \frac{16}{105},\ c_3 = \frac{8}{35},\ c_4 = \frac{9}{20},\ c_5 = \frac{2}{3},\ c_6 = \frac{7}{9},\ c_7 = 1.$$

The principal error norm of the order 5 scheme, that is, the 2-norm of the principal error terms is: $0.7055529138 \times 10^{(-4)}$.

The 2-norm of the order 7 error terms is $0.1774339540 \times 10^{(-3)}$, which is about 2.515 times the previous norm.

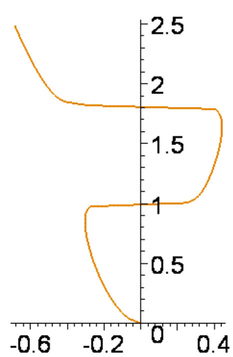
The principal error norm of the order 4 embedded scheme is: $0.7814366417 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 0.8582519531.
The 2-norm of the linking coefficients is: 1.982535647.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.9157, 0]$ and $[-4.7749, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0.9970, 1.8195].

The Butcher tableau of the combined scheme is as follows.

$$
\begin{bmatrix}
\frac{16}{105} & \frac{16}{105} & & & & & & \\
\frac{8}{35} & \frac{2}{35} & \frac{6}{35} & & & & & \\
\frac{9}{20} & \frac{8793}{40960} & -\frac{5103}{8192} & \frac{17577}{20480} & & & & \\
\frac{2}{3} & \frac{347}{1458} & -\frac{7}{20} & \frac{3395}{10044} & \frac{49792}{112995} & & & \\
\frac{7}{9} & -\frac{1223224109959}{9199771214400} & \frac{1234787701}{2523942720} & \frac{568994101921}{3168810084960} & -\frac{105209683888}{891227836395} & \frac{9}{25} & & \\
1 & \frac{2462504862877}{8306031988800} & -\frac{123991}{287040} & \frac{106522578491}{408709510560} & \frac{590616498832}{804646848915} & -\frac{319138726}{534081275} & \frac{52758}{71449} & \\
b & \frac{1093}{15120} & 0 & \frac{60025}{190992} & \frac{3200}{20709} & \frac{1611}{11960} & \frac{712233}{2857960} & \frac{3}{40} \\
b* & \frac{84018211}{991368000} & 0 & \frac{92098979}{357791680} & \frac{17606944}{67891005} & \frac{3142101}{235253200} & \frac{22004596809}{70270091500} & \frac{9}{125}
\end{bmatrix}
$$

The coefficients are as follows:
c[2]=16/105,
c[3]=8/35,
c[4]=9/20,
c[5]=2/3,
c[6]=7/9,
c[7]=1,

a[2,1]=16/105,
a[3,1]=2/35,
a[3,2]=6/35,
a[4,1]=8793/40960,
a[4,2]=-5103/8192,
a[4,3]=17577/20480,
a[5,1]=347/1458,
a[5,2]=-7/20,
a[5,3]=3395/10044,
a[5,4]=49792/112995,
a[6,1]=-1223224109959/9199771214400,
a[6,2]=1234787701/2523942720,
a[6,3]=568994101921/3168810084960,
a[6,4]=-105209683888/891227836395,
a[6,5]=9/25,
a[7,1]=2462504862877/8306031988800,
a[7,2]=-123991/287040,
a[7,3]=106522578491/408709510560,
a[7,4]=590616498832/804646848915,
a[7,5]=-319138726/534081275,
a[7,6]=52758/71449,

b[1]=1093/15120,
b[2]=0,
b[3]=60025/190992,
b[4]=3200/20709,
b[5]=1611/11960,
b[6]=712233/2857960,
b[7]=3/40,

b*[1]=84018211/991368000,
b*[2]=0,
b*[3]=92098979/357791680,
b*[4]=17606944/67891005,
b*[5]=3142101/235253200,
b*[6]=22004596809/70270091500,
b*[7]=9/125.

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