A combined 7 stage, order 4 and 5 Runge-Kutta scheme with an additional 8 stage, order 4 FSAL embedded scheme

See: An Efficient Runge-Kutta (4,5) Pair by P.Bogacki and L.F.Shampine
Computers and Mathematics with Applications, Vol. 32, No. 6, 1996, pages 15 to 28

The scheme is constructed so that the stability radius is approximately equal to the imaginary axis inclusion.
The nodes of this scheme are as follows.

$$c_2 = \frac{11}{63},\ c_3 = \frac{4}{17},\ c_4 = \frac{174}{349},\ c_5 = \frac{36}{49},\ c_6 = \frac{97}{129},\ c_7 = 1,\ c_8 = 1.$$

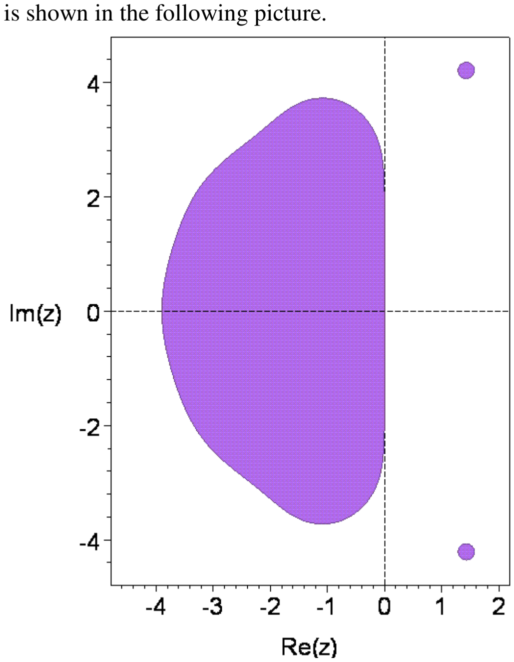
The principal error norm of the order 5 scheme, that is, the 2-norm of the principal error terms is: $0.1976847989 \times 10^{(-4)}$.

The principal error norm of the 7 stage, order 4 embedded scheme is: $0.9439150376 \times 10^{(-4)}$.

The principal error norm of the 8 stage, order 4 embedded scheme is: $0.9465487457 \times 10^{(-4)}$.

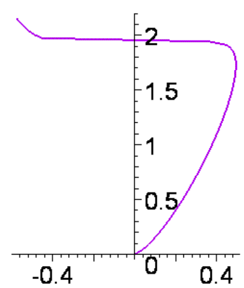
The maximum magnitude of the linking coefficients is: 1.745655938.
The 2-norm of the linking coefficients is: 3.167684916.

The stability region for the order 5 scheme is shown in the following picture.



The real stability interval of the order 5 scheme is $[-3.8951, 0]$.
The stability regions for the two embedded order 4 schemes are very similar in size and shape to that of the order 4 scheme.
The real stability intervals for the 7 stage and 8 stage, order 4 schemes are respectively $[-3.9689, 0]$ and $[-3.9265, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[0, 1.9454]$.

The coefficients are as follows:
c[2]=11/63,
c[3]=4/17,
c[4]=174/349,
c[5]=36/49,
c[6]=97/129,
c[7]=1,
c[8]=1,

a[2,1]=11/63,
a[3,1]=244/3179,
a[3,2]=504/3179,
a[4,1]=184816101/935188078,
a[4,2]=-411995808/467594039,
a[4,3]=100493613/85017098,
a[5,1]=2424510108/16025323237,
a[5,2]=12241368/78942479,
a[5,3]=-4360301712/39234412063,
a[5,4]=613806882624/1137797949827,
a[6,1]=166741171875227/3452798773601100,
a[6,2]=1006084/2623731,
a[6,3]=-188382845675879/1408900648998150,
a[6,4]=89559834743712383/245148712925678100,
a[6,5]=31/350,
a[7,1]=6022012446846011204537694339419654322731/30099864007383356992178712977537163264000,
a[7,2]=-6561/8296,
a[7,3]=341104813140161009375674013573715202837301/313319409731603971335732895410416320512000,
a[7,4]=95341943376640595142991543684079745176400281/1318406651709399034894169155355186749556736000,
a[7,5]=-1154594845209196567962511465161858938950699/877446764516774644979830270188996722688000,
a[7,6]=1085508425299375/621834120640512,
a[8,1]=4827113239/65475838080,
a[8,2]=0,
a[8,3]=797791005101/2404074449920,
a[8,4]=5703219796216031/30660055749940320,
a[8,5]=-33901013357491/177576803988480,
a[8,6]=29204563953141/55776754401170,
a[8,7]=34/449,

b[1]=4827113239/65475838080,
b[2]=0,
b[3]=797791005101/2404074449920,
b[4]=5703219796216031/30660055749940320,
b[5]=-33901013357491/177576803988480,
b[6]=29204563953141/55776754401170,
b[7]=34/449,

b^[1]=7345858582709/98987482023552,
b^[2]=0,
b^[3]=30485818726152979/92415623446907904,
b^[4]=2471755716006328519/13007276847211309344,
b^[5]=-5627835948231581/26545332727174144,
b^[6]=45/83,
b^[7]=34/449,

b*[1]=66873629126427899/902163023828283336,
b*[2]=0,
b*[3]=121/366,
b*[4]=3468482659797555651329/18566189507645514457800,
b*[5]=-65/379,
b*[6]=952822237714091128053/1892592848332731588736,
b*[7]=129752050156606597/1891022063549226300,
b*[8]=568662239802085502227/7201012017995453750400.

Version: 21 Dec 2012, Peter Stone