A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

See: Runge–Kutta pairs of orders 5(4) using the minimal set of simplifying assumptions,
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The nodes of the scheme excluding c_5 are:

$$c_2 = \frac{42}{277},\; c_3 = \frac{256}{767},\; c_4 = \frac{20}{21},\; c_6 = 1,\; c_7 = 1.$$

c_5 is a rational function of the linking coefficient $a_{3,2}$ which, in turn, is a real zero of a degree 6 polynomial that depends on the nodes c_2, c_3 and c_4 together with the weight $b_2 = \frac{13}{1000}$.

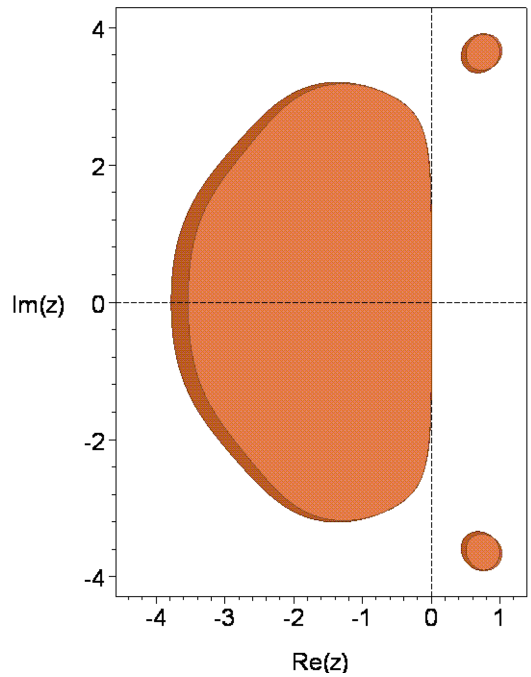
The principal error norm, that is, the 2-norm of the principal error terms is: $0.8565288328 \times 10^{(-4)}$.

The principal error norm of the order 4 embedded scheme is: $0.6593434488 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 21.19859369.

The 2-norm of the linking coefficients is: 35.27712684.

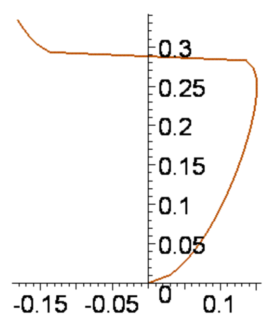
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.

The real stability intervals of the order 5 and 4 schemes are respectively $[-3.5370, 0]$ and $[-3.7818, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.2883].

The coefficients correct to 85 digits are as follows:
c[2]=.1516245487364620938628158844765342960288808664259927797833935018050541516245487364621,
c[3]=.3337679269882659713168187744458930899608865710560625814863102998696219035202086049544,
c[4]=.9523809523809523809523809523809523809523809523809523809523809523809523809523809523810,
c[5]=.9918664704001919171492864877384488842517040385273769447369625194272375660818507995855,
c[6]=1.,
c[7]=1.,

a[2,1]=.1516245487364620938628158844765342960288808664259927797833935018050541516245487364621,
a[3,1]=-.3783529569947882308924177902112930260431004372161816404973593684979893414027309091685e-1,
a[3,2]=.3716032226877447944060605534670223925651966147776807455360462367194208376604816958712,
a[4,1]=4.088241688432284570741178114488772648015205516346455044561659157821025407222276668928,
a[4,2]=-8.217782247903120506027186724157853205297550999655206279593000003466433846460086276845,
a[4,3]=5.081921511851788316238389562050032938234726435689703615983721798026360820190190560298,
a[5,1]=5.314402695391048568216621976814253482588658155313799723483257549295195540903641590806,
a[5,2]=-10.69519821306241324657564217375648822971476297384037057001265855043518132853896359137,
a[5,3]=6.401095800769576826498066335770884194828684349483685057184293937302789829167759282646,
a[5,4]=-.2843381269802023098975965109020056345087549242973726591793041673556647545058648249462e-1,
a[6,1]=5.563118224853566200838153383514333560634082269602834496737017588814594584832825727575,
a[6,2]=-11.19522888819289884981282780323268974734268225535804287062729844368290245931900604516,
a[6,3]=6.664456762398461339227359710069845295832439575601365272488290561052516437818636482404,
a[6,4]=-.2195278706834865571537529353492219629161069576768152262399119561814696002292252636108e-1,
a[6,5]=-.1039331199078003453730999681656691283222889407847537597401851056606160330953363846235e-1,
a[7,1]=.9684360647515179773331790267425702404478600996174168459686049441630450181586822539173e-1,
a[7,2]=.13e-1,
a[7,3]=.4892812311240123225593586884864013024706358286409068500907631350142440263587658492382,
a[7,4]=5.010013112528675392237820086324100789769601503713903121290133640375239377010718020729,
a[7,5]=-21.19859368565969502065831857210633964423602520905909596590625359433714896552353829162,
a[7,6]=16.58945573553185550812782189462158052795100186674254430992849632453136106033818619626,

b[1]=.9684360647515179773331790267425702404478600996174168459686049441630450181586822539173e-1,
b[2]=.13e-1,
b[3]=.4892812311240123225593586884864013024706358286409068500907631350142440263587658492382,
b[4]=5.010013112528675392237820086324100789769601503713903121290133640375239377010718020729,
b[5]=-21.19859368565969502065831857210633964423602520905909596590625359433714896552353829162,
b[6]=16.58945573553185550812782189462158052795100186674254430992849632453136106033818619626,

b*[1]=.9561524921911732442029505775904650412709156045280126480771123555202374993286796444779e-1,
b*[2]=.1244623980789377910387623115026107792586306718449540180034135858069444324561903013951e-1,
b*[3]=.4947247631578494883255918983651363130191122539285464661110617378564151958122161886503,
b*[4]=4.598587821619537776155564168914626088719652833440427613102620444638793422279346889390,
b*[5]=-19.02694379120369196716736067720516482188264060983689214139814898152600347391922363987,
b*[6]=14.81699828882786502773346189244466626666234946625919282414784277632664809122060213867,
b*[7]=.8571428571428571428571428571428571428571428571428571428571428571428571428571428571429e-2.

Version: 19 Dec 2012, Peter Stone