A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

See: Runge–Kutta pairs of orders 5(4) using the minimal set of simplifying assumptions,
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The nodes of the scheme excluding c_5 are:

$$c_2 = \frac{40}{261},\ c_3 = \frac{169}{508},\ c_4 = \frac{33}{35},\ c_6 = 1,\ c_7 = 1.$$

c_5 is a rational function of the linking coefficient $a_{3,2}$ which, in turn, is a real zero of a degree 6 polynomial that depends on the nodes c_2, c_3 and c_4 together with the weight $b_2 = \frac{1}{80}$.

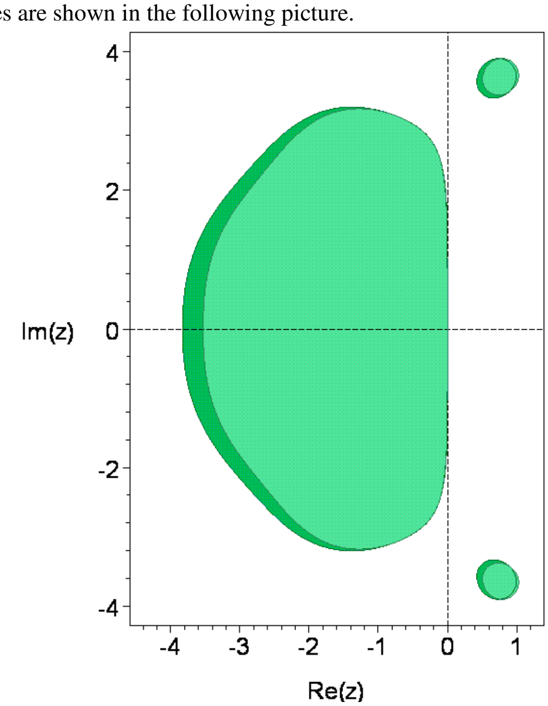
The principal error norm, that is, the 2-norm of the principal error terms is: $0.9387796436 \times 10^{(-4)}$.

The principal error norm of the order 4 embedded scheme is: $0.7589554491 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 14.43385367.

The 2-norm of the linking coefficients is: 29.12905307.

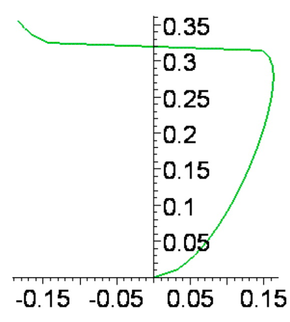
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.

The real stability intervals of the order 5 and 4 schemes are respectively $[-3.5330, 0]$ and $[-3.8321, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.3209].

The coefficients correct to 85 digits are as follows:
c[2]=.1532567049808429118773946360153256704980842911877394636015325670498084291187739463602,
c[3]=.3326771653543307086614173228346456692913385826771653543307086614173228346456692913386,
c[4]=.9428571428571428571428571428571428571428571428571428571428571428571428571428571428571,
c[5]=.9905291481600554062610492983695588299582582619956845560775353294408404384227117473986,
c[6]=1.,
c[7]=1.,

a[2,1]=.1532567049808429118773946360153256704980842911877394636015325670498084291187739463602,
a[3,1]=-.3251061810785323309893943849954503841690499714388494058880708816352891082641438283267e-1,
a[3,2]=.3651877834621839417603567613341907077082435798210502949195157495808517454720836741713,
a[4,1]=3.848600657232453282264542278770732468565471035613008665862084564211836811283455082049,
a[4,2]=-7.847387984332609177818183708487289941915918946487415460180213670812729536087599756166,
a[4,3]=4.941644469957298752696498572573700330493305053731549651460986249458035581947001816974,
a[5,1]=5.318773612671963348193642428150166262217164708231686557641944531517881990371774788250,
a[5,2]=-10.86300018934432190483846169063207331192222298712977344674490146155482360751928982050,
a[5,3]=6.571982836285947414768066366505769935928586808323342154078675660438079350323131075366,
a[5,4]=-.3722711145353345186219780565430405626527026742957070889818340096029729475290429571547e-1,
a[6,1]=5.603718187418808325128679217449782316501237632766850710291397504180102488473553836201,
a[6,2]=-11.44386123388941550926228700849166854616646956243562387682584167133036252315098421035,
a[6,3]=6.881865914052513803700577459499137731660837144860179172244831612516044580797808314633,
a[6,4]=-.2959332255078882070989817187349712937155702416885373060083027546863209016942983180889e-1,
a[6,5]=-.1212954503111779885707149658375437262404819102255227510955716989715245595094810867443e-1,
a[7,1]=.9680779501172470861958460213329129633333950431916213995260523180795958940505926325063e-1,
a[7,2]=.125e-1,
a[7,3]=.4877655758611665143380669647542063097753103171071291760445045750715447796688219311173,
a[7,4]=3.566704692175309228914265864609715268530335626436188172329830573535260761321172728469,
a[7,5]=-14.43385367353046213668503285020783586494860625770896072631531627942409414513169129955,
a[7,6]=11.27007561048226168481311541871062299030962080984648123798837589900932901473663737672,

b[1]=.9680779501172470861958460213329129633333950431916213995260523180795958940505926325063e-1,
b[2]=.125e-1,
b[3]=.4877655758611665143380669647542063097753103171071291760445045750715447796688219311173,
b[4]=3.566704692175309228914265864609715268530335626436188172329830573535260761321172728469,
b[5]=-14.43385367353046213668503285020783586494860625770896072631531627942409414513169129955,
b[6]=11.27007561048226168481311541871062299030962080984648123798837589900932901473663737672,

b*[1]=.9541482098477637533552978070277860897718645955307682493279267788746488496270312205507e-1,
b*[2]=.1186786967074662789158439213492202020551271210706199249601473797950607417458393045977e-1,
b*[3]=.4939432182124876669775144789142199056807020719259743090677406925768600006338504598547,
b*[4]=3.242177372881165105283149968629298281133631418293171591166274874794315666872425781430,
b*[5]=-12.70748704239684440346553469655168712303017001818356150247047048098437518766613099155,
b*[6]=9.854083760647668627977756076170468307033137356304276784807647497746228561022567697751,
b*[7]=.1e-1.

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