

Tsitouras' (2011) 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

See: Runge–Kutta pairs of orders 5(4) using the minimal set of simplifying assumptions,
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The nodes of the scheme excluding c_5 are:

$$c_2 = \frac{161}{1000}, c_3 = \frac{327}{1000}, c_4 = \frac{9}{10}, c_6 = 1, c_7 = 1.$$

c_5 is a rational function of the linking coefficient $a_{3,2}$ which, in turn, is a real zero of a degree 6 polynomial that depends on the nodes c_2 , c_3 and c_4 together with the weight $b_2 = \frac{1}{100}$.

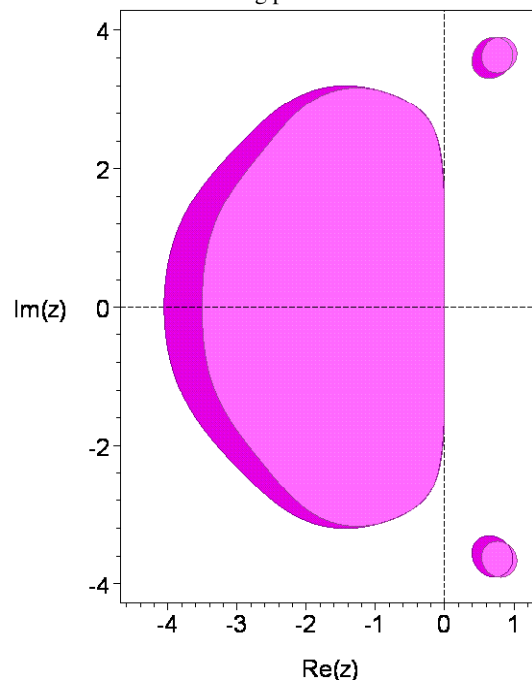
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1385149964 \times 10^{(-3)}$.

The principal error norm of the order 4 embedded scheme is: $0.1064972833 \times 10^{(-2)}$.

The maximum magnitude of the linking coefficients is: 12.92096932.

The 2-norm of the linking coefficients is: 24.01783764.

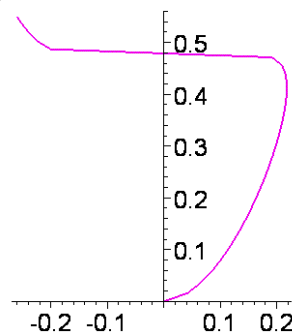
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.

The real stability intervals of the order 5 and 4 schemes are respectively $[-3.5068, 0]$ and $[-4.0560, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[0, 0.4780]$.

