A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation, Volume 65, Number 215, July 1996, Pages 1165-1181.

Note: The order 5 scheme satisfies 7 of the 20 principal error conditions.

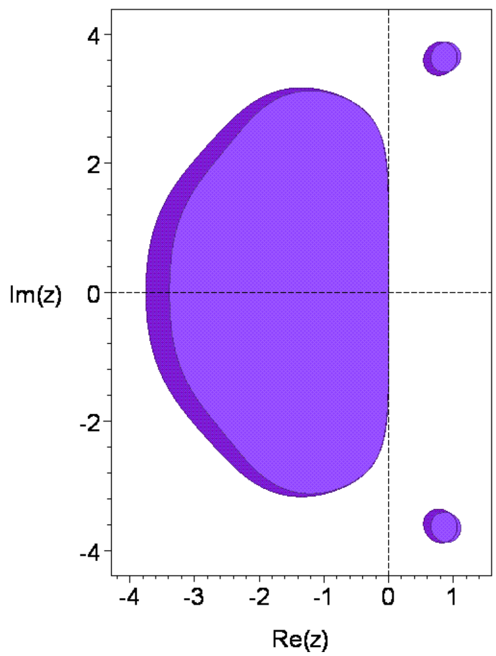
The nodes of the scheme are:

$$c_2 = \frac{41}{288},\ c_3 = \frac{5}{16},\ c_4 = \frac{6}{7},\ c_5 = \frac{33}{35},\ c_6 = 1,\ c_7 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.2721519359 \times 10^{(-3)}$.

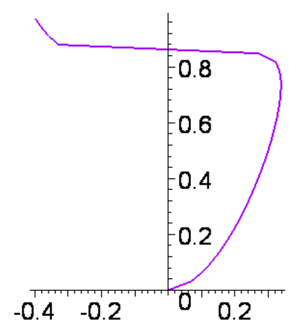
The principal error norm of the order 4 embedded scheme is: $0.7914997787 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 16.89455910.
The 2-norm of the linking coefficients is: 28.73236849.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.3865, 0]$ and $[-3.7519, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.8523].

The Butcher tableau is as follows.

$$\begin{bmatrix} \frac{41}{288} & \frac{41}{288} & & & & & \\ \frac{5}{16} & -\frac{5}{164} & \frac{225}{656} & & & & \\ \frac{6}{7} & \frac{159402}{50225} & -\frac{451008}{70315} & \frac{35136}{8575} & & & \\ \frac{33}{35} & \frac{653199723}{95427500} & -\frac{464374944}{33399625} & \frac{2023011936}{248460625} & -\frac{81543}{579500} & & \\ 1 & \frac{36045973}{4317300} & -\frac{45024}{2665} & \frac{608961056}{62984025} & \frac{539}{256932} & -\frac{46550}{371709} & \\ 1 & \frac{1123}{11880} & 0 & \frac{557056}{1184315} & \frac{12691}{13176} & -\frac{814625}{838728} & \frac{39}{88} \\ b & \frac{1123}{11880} & 0 & \frac{557056}{1184315} & \frac{12691}{13176} & -\frac{814625}{838728} & \frac{39}{88} \\ b^* & \frac{1904827}{20433600} & 0 & \frac{120907648}{254627725} & \frac{20518603}{22662720} & -\frac{247483075}{288522432} & \frac{56277}{151360} & \frac{1}{80} \end{bmatrix}$$

The coefficients are as follows:
c[2]=41/288,
c[3]=5/16,
c[4]=6/7,
c[5]=33/35,
c[6]=1,
c[7]=1,

a[2,1]=41/288,
a[3,1]=-5/164,
a[3,2]=225/656,
a[4,1]=159402/50225,
a[4,2]=-451008/70315,
a[4,3]=35136/8575,
a[5,1]=653199723/95427500,
a[5,2]=-464374944/33399625,
a[5,3]=2023011936/248460625,
a[5,4]=-81543/579500,
a[6,1]=36045973/4317300,
a[6,2]=-45024/2665,
a[6,3]=608961056/62984025,
a[6,4]=539/256932,
a[6,5]=-46550/371709,
a[7,1]=1123/11880,
a[7,2]=0,
a[7,3]=557056/1184315,
a[7,4]=12691/13176,
a[7,5]=-814625/838728,
a[7,6]=39/88,

b[1]=1123/11880,
b[2]=0,
b[3]=557056/1184315,
b[4]=12691/13176,
b[5]=-814625/838728,
b[6]=39/88,

b*[1]=1904827/20433600,
b*[2]=0,
b*[3]=120907648/254627725,
b*[4]=20518603/22662720,
b*[5]=-247483075/288522432,
b*[6]=56277/151360,
b*[7]=1/80.

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