A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation,Volume 65, Number 215, July 1996, Pages 1165-1181.

Note: The order 5 scheme satisfies 7 of the 20 principal error conditions.

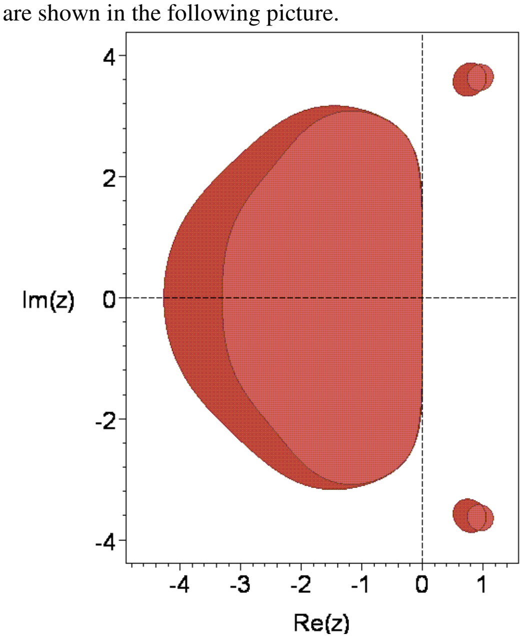
The nodes of the scheme are:

$$c_2 = \frac{3}{20},\ c_3 = \frac{3}{10},\ c_4 = \frac{4}{5},\ c_5 = \frac{19}{21},\ c_6 = 1,\ c_7 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.4195197107 \times 10^{(-3)}$.

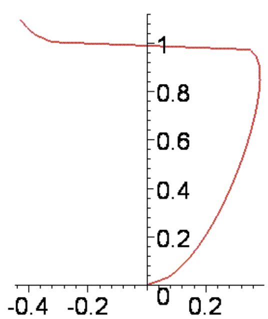
The principal error norm of the order 4 embedded scheme is: $0.1253553582 \times 10^{(-2)}$.
The maximum magnitude of the linking coefficients is: 17.82624867.
The 2-norm of the linking coefficients is: 31.62863021.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-3.3066, 0]$ and $[-4.2704, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.9972].

The Butcher tableau is as follows.

$$\left[\begin{array}{cccccccc}
\frac{3}{20} & \frac{3}{20} & & & & & & \\
\frac{3}{10} & 0 & \frac{3}{10} & & & & & \\
\frac{4}{5} & \frac{20}{9} & -\frac{224}{45} & \frac{32}{9} & & & & \\
\frac{19}{21} & \frac{5238623}{666792} & -\frac{1485800}{83349} & \frac{936244}{83349} & -\frac{26543}{74088} & & & \\
1 & \frac{257789}{34200} & -\frac{152}{9} & \frac{298732}{28575} & \frac{203}{2200} & -\frac{129654}{663575} & & \\
1 & \frac{83}{912} & 0 & \frac{400}{889} & \frac{325}{528} & -\frac{64827}{212344} & \frac{25}{168} & \\
b & \frac{83}{912} & 0 & \frac{400}{889} & \frac{325}{528} & -\frac{64827}{212344} & \frac{25}{168} & \\
b* & \frac{5153}{57456} & 0 & \frac{76360}{168021} & \frac{6445}{11088} & -\frac{53655}{212344} & \frac{365}{3528} & \frac{1}{42}
\end{array}\right]$$

The coefficients are as follows:
c[2]=3/20,
c[3]=3/10,
c[4]=4/5,
c[5]=19/21,
c[6]=1,
c[7]=1,

a[2,1]=3/20,
a[3,1]=0,
a[3,2]=3/10,
a[4,1]=20/9,
a[4,2]=-224/45,
a[4,3]=32/9,
a[5,1]=5238623/666792,
a[5,2]=-1485800/83349,
a[5,3]=936244/83349,
a[5,4]=-26543/74088,
a[6,1]=257789/34200,
a[6,2]=-152/9,
a[6,3]=298732/28575,
a[6,4]=203/2200,
a[6,5]=-129654/663575,
a[7,1]=83/912,
a[7,2]=0,
a[7,3]=400/889,
a[7,4]=325/528,
a[7,5]=-64827/212344,
a[7,6]=25/168,

b[1]=83/912,
b[2]=0,
b[3]=400/889,
b[4]=325/528,
b[5]=-64827/212344,
b[6]=25/168,

b*[1]=5153/57456,
b*[2]=0,
b*[3]=76360/168021,
b*[4]=6445/11088,
b*[5]=-53655/212344,
b*[6]=365/3528,
b*[7]=1/42.

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