A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation, Volume 65, Number 215, July 1996, Pages 1165-1181.

Note: The order 5 scheme is constructed so that the stability function is exactly

$$R(z) = 1 + z + \frac{1}{2} z^2 + \frac{1}{6} z^3 + \frac{1}{24} z^4 + \frac{1}{120} z^5 + \frac{1}{1296} z^6.$$

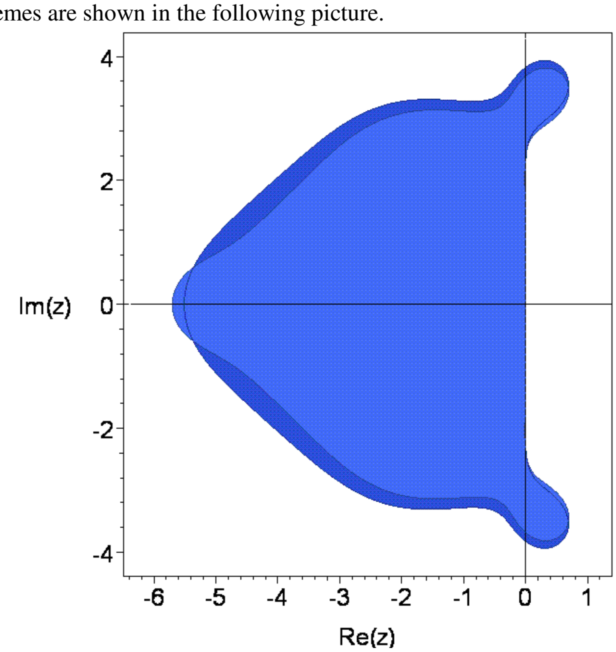
This ensures that the stability region is the same as that of a scheme of Prince and Dormand.

The nodes of the scheme are:

$$c_2 = \frac{43}{207},\ c_3 = \frac{44}{137},\ c_4 = \frac{685}{1458},\ c_5 = \frac{23}{40},\ c_6 = 1,\ c_7 = 1.$$

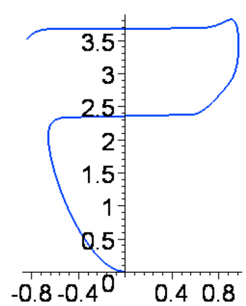
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1688966379 \times 10^{(-2)}$.
The principal error norm of the order 4 embedded scheme is: $0.4789152663 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 8.452499350.
The 2-norm of the linking coefficients is: 10.98234016.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-5.7046, 0]$ and $[-5.5111, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[2.3504, 3.6804]$.

The coefficients are as follows:
c[2]=43/207,
c[3]=44/137,
c[4]=685/1458,
c[5]=23/40,
c[6]=1,
c[7]=1,

a[2,1]=43/207,
a[3,1]=58828/807067,
a[3,2]=200376/807067,
a[4,1]=28959683334245/258015846946176,
a[4,2]=25717753025/651555169056,
a[4,3]=1908779615725/6000368533632,
a[5,1]=3036710412230543/17558188651520000,
a[5,2]=-25228364913/116510873600,
a[5,3]=9564831652720903/17700060961792000,
a[5,4]=61215263996679/782834566040000,
a[6,1]=-3753979488587/6205470251552,
a[6,2]=1184924097/1029440984,
a[6,3]=15239890896624334 3/43507775962901536,
a[6,4]=-28689560422600158/3394210307911397,
a[6,5]=368915177600/68271950591,
a[7,1]=3263/45210,
a[7,2]=0,
a[7,3]=40863941876/42252931149,
a[7,4]=-75502829419254/48221189556655,
a[7,5]=18112000/12798591,
a[7,6]=16003/143778,

b[1]=3263/45210,
b[2]=0,
b[3]=40863941876/42252931149,
b[4]=-75502829419254/48221189556655,
b[5]=18112000/12798591,
b[6]=16003/143778,

b*[1]=1218193003/14093313300,
b*[2]=0,
b*[3]=64542669378056333/79029037362466620,
b*[4]=-3126658200891487254/2505331903416010525,
b*[5]=488928006400/398970477243,
b*[6]=6532856681/67229873910,
b*[7]=1/45.

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