A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation, Volume 65, Number 215, July 1996, Pages 1165-1181.

Note: The stability region of the order 5 scheme has its stability radius equal to its imaginary axis inclusion.
The nodes of the scheme are:

$$c_2 = \frac{17}{150},\ c_3 = \frac{230}{1379},\ c_4 = \frac{1420370}{2342387},\ c_5 = \frac{133}{208},\ c_6 = 1,\ c_7 = 1.$$

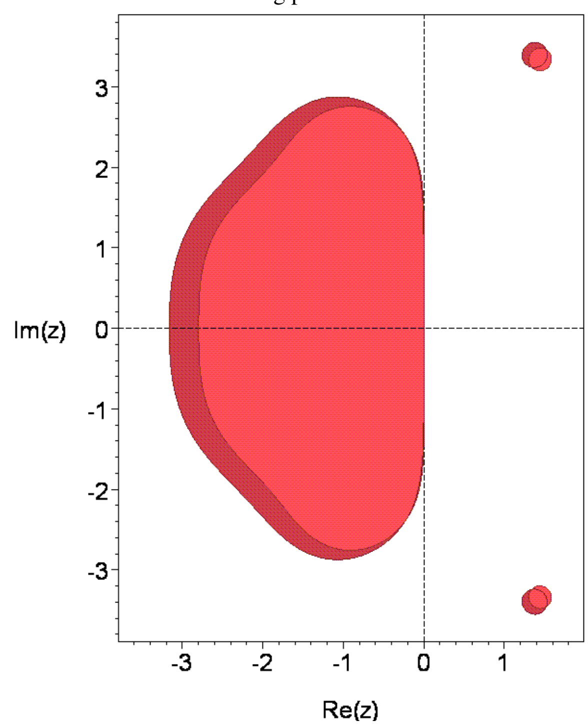
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2295487133 \times 10^{(-2)}$.

The principal error norm of the order 4 embedded scheme is: $0.1134515387 \times 10^{(-2)}$.

The maximum magnitude of the linking coefficients is: 6.928725281.

The 2-norm of the linking coefficients is: 10.23635319.

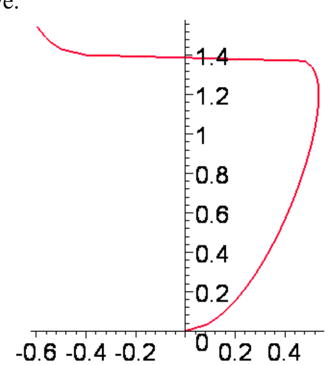
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.

The real stability intervals of the order 5 and 4 schemes are respectively $[-2.7960, 0]$ and $[-3.1628, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[0, 1.3980]$.

The coefficients are as follows:
c[2]=17/150,
c[3]=230/1379,
c[4]=1420370/2342387,
c[5]=133/208,
c[6]=1,
c[7]=1,

a[2,1]=17/150,
a[3,1]=1424390/32327897,
a[3,2]=3967500/32327897,
a[4,1]=11725892832745653435650/6798789880492106984987,
a[4,2]=-2048128144537681132500/295599560021395955869,
a[4,3]=39503685458474301158220/6798789880492106984987,
a[5,1]=12161603119957121528213173/25732152506856316150988800,
a[5,2]=-1141151413305973125/551372154861039616,
a[5,3]=3683267576399581353585064593/1765398936832403041633484800,
a[5,4]=5081806128006459902107790523/33857948797038081396772249600,
a[6,1]=-778166632716677780702867/2973121469222107001182300,
a[6,2]=104590105895625/68427659422081,
a[6,3]=-6240811551117087470432064137/6600414167621042866395511800,
a[6,4]=-963956247541399247752708263264927/631428144652436311985040262094600,
a[6,5]=4149774798413464954572800/1881639624293239507076793,
a[7,1]=10715793331/868982366000,
a[7,2]=0,
a[7,3]=380753067446365335491/1017424549568194596000,
a[7,4]=-33988229203965941757637160569/199599379574494873119383412000,
a[7,5]=73328266119626752/108862061490007875,
a[7,6]=175006801591/1589096299500,

b[1]=10715793331/868982366000,
b[2]=0,
b[3]=380753067446365335491/1017424549568194596000,
b[4]=-33988229203965941757637160569/199599379574494873119383412000,
b[5]=73328266119626752/108862061490007875,
b[6]=175006801591/1589096299500,

b*[1]=36166679554284109/3035692917188954400,
b*[2]=0,
b*[3]=1333442845934451852070878169/3554259119336756010609086400,
b*[4]=-372495557697385192340734287120958393/2091834471758211976844177479899662400,
b*[5]=129508101849060942811648/190148731404640113216825,
b*[6]=58914464694395831/693916322394528225,
b*[7]=1/40.

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