A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation,Volume 65, Number 215, July 1996, Pages 1165-1181.

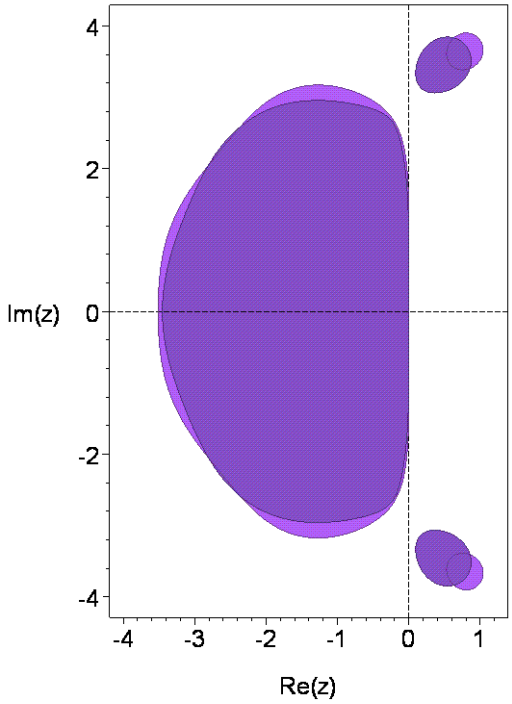
The nodes of the scheme are:

$$c_2 = \frac{1}{5},\ c_3 = \frac{17}{52},\ c_4 = \frac{15}{16},\ c_5 = \frac{63}{64},\ c_6 = 1,\ c_7 = 1.$$

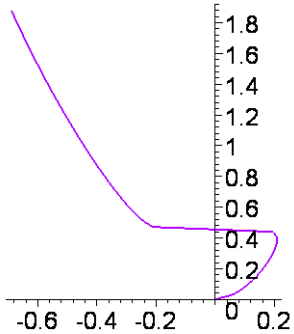
The principal error norm, that is, the 2-norm of the principal error terms is: $0.7565762051 \times 10^{(-4)}$.

The principal error norm of the order 4 embedded scheme is: $0.3575692995 \times 10^{(-2)}$.
The maximum magnitude of the linking coefficients is: 9.045256301.
The 2-norm of the linking coefficients is: 21.40541341.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively [−3.5120, 0] and [−3.4526, 0].
The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.4522].

The coefficients are as follows:
c[2]=1/5,
c[3]=17/52,
c[4]=15/16,
c[5]=63/64,
c[6]=1,
c[7]=1,
a[2,1]=1/5,
a[3,1]=323/5408,
a[3,2]=1445/5408,
a[4,1]=570165/295936,
a[4,2]=-104625/17408,
a[4,3]=371475/73984,
a[5,1]=26934500313/10132848640,
a[5,2]=-990580185/119209984,
a[5,3]=429391882647/64343588864,
a[5,4]=-654759/17393920,
a[6,1]=288895547/99769158,
a[6,2]=-655365/72454,
a[6,3]=923373618440/128349326613,
a[6,4]=-156352/7307199,
a[6,5]=-2191360/94418217,
a[7,1]=47483/481950,
a[7,2]=0,
a[7,3]=102134136/206670275,
a[7,4]=288256/85725,
a[7,5]=-14024704/1550745,
a[7,6]=2131/350,

b[1]=47483/481950,
b[2]=0,
b[3]=102134136/206670275,
b[4]=288256/85725,
b[5]=-14024704/1550745,
b[6]=2131/350,

b*[1]=230978806/2557949625,
b*[2]=0,
b*[3]=1145778873174/2193804969125,
b*[4]=1713455584/909970875,
b*[5]=-62703794176/16461158175,
b*[6]=16819983/7430500,
b*[7]=1/20.

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