A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

This scheme is constructed using an algorithm of S.N. Papakostas and G. PapaGeorgiou.
See: A Family of Fifth-order Runge-Kutta Pairs, by S.N. Papakostas and G. PapaGeorgiou,
Mathematics of Computation,Volume 65, Number 215, July 1996, Pages 1165-1181.

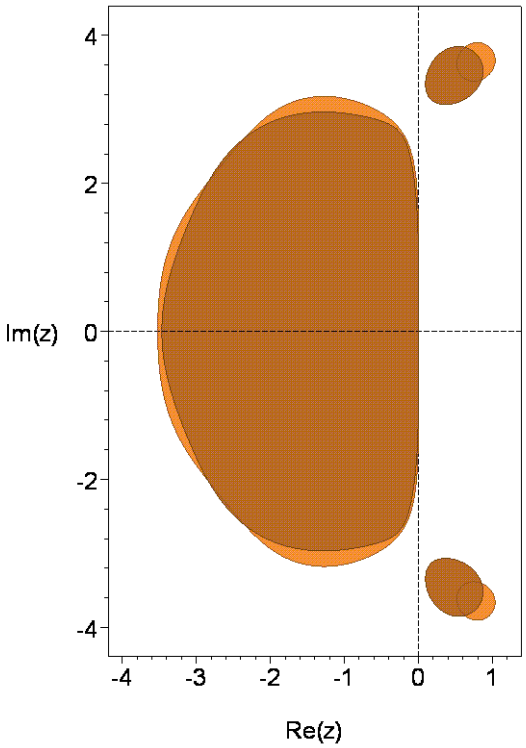
The nodes of the scheme are:

$$c_2 = \frac{14}{87},\ c_3 = \frac{22}{67},\ c_4 = \frac{19}{20},\ c_5 = \frac{90}{91},\ c_6 = 1,\ c_7 = 1.$$

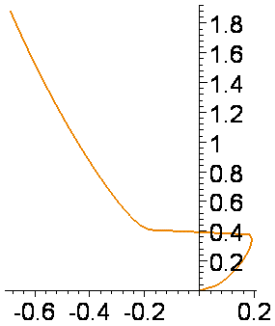
The principal error norm, that is, the 2-norm of the principal error terms is: $0.5753048304 \times 10^{(-4)}$.

The principal error norm of the order 4 embedded scheme is: $0.3804792045 \times 10^{(-2)}$.
The maximum magnitude of the linking coefficients is: 16.20753109.
The 2-norm of the linking coefficients is: 29.66455169.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively [−3.5218, 0] and [−3.4666, 0].
The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval [0, 0.3968].

The coefficients are as follows:
c[2]=14/87,
c[3]=22/67,
c[4]=19/20,
c[5]=90/91,
c[6]=1,
c[7]=1,

a[2,1]=14/87,
a[3,1]=-209/31423,
a[3,2]=10527/31423,
a[4,1]=24122951/6776000,
a[4,2]=-19252491/2464000,
a[4,3]=20147771/3872000,
a[5,1]=30735990279/6656292643,
a[5,2]=-6137182971/605117513,
a[5,3]=36273048382476/5544691771619,
a[5,4]=-1300492800/45823898939,
a[6,1]=499359810/101691667,
a[6,2]=-20986575/1946252,
a[6,3]=35434847017431/5130954750152,
a[6,4]=-130147200/7100767723,
a[6,5]=-55010683/3614316344,
a[7,1]=44623/451440,
a[7,2]=0,
a[7,3]=19808551943/39861249120,
a[7,4]=16552000/3371151,
a[7,5]=-5005972153/308867040,
a[7,6]=6319/540,

b[1]=44623/451440,
b[2]=0,
b[3]=19808551943/39861249120,
b[4]=16552000/3371151,
b[5]=-5005972153/308867040,
b[6]=6319/540,

b*[1]=5097401/56517780,
b*[2]=0,
b*[3]=1313519543033/2495203469220,
b*[4]=39381014000/15193777557,
b*[5]=-21509177053/3222369790,
b*[6]=398097/90140,
b*[7]=1/20.

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