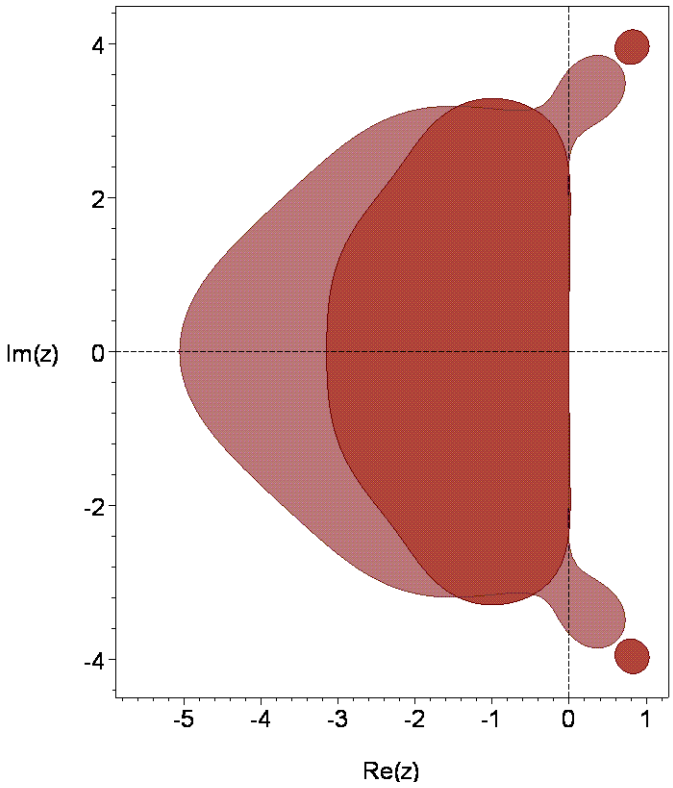
A 6 stage, combined order 4 and 5 Runge-Kutta scheme

The scheme considered here was constructed to have (near) maximal stability radius.
The nodes of the scheme are:

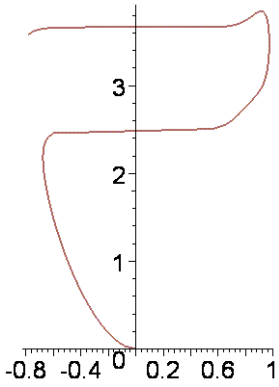
$$c_2 = \frac{5}{19},\ c_3 = \frac{37}{126},\ c_4 = \frac{2331}{4073},\ c_5 = \frac{5}{6},\ c_6 = 1.$$

The principal error norm, that is, the 2-norm of the principal error terms is: $0.1983864954 \times 10^{(-2)}$.
The principal error norm of the order 4 embedded scheme is: $0.1679408046 \times 10^{(-2)}$.
The maximum magnitude of the linking coefficients is: 11.04552015.
The 2-norm of the linking coefficients is: 16.84412442.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively $[-5.0571, 0]$ and $[-4.7745, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: $[2.4923, 3.6640]$.

TheButcher tableau of the scheme is as follows.

$$\begin{bmatrix}
\frac{12}{41} & \frac{12}{41} & & & & & \\
\frac{53}{151} & \frac{76903}{547224} & \frac{115169}{547224} & & & & \\
\frac{8003}{9668} & \frac{2011866719207}{10844041507584} & -\frac{21792911494331}{10844041507584} & \frac{2396462600599}{903670125632} & & & \\
\frac{7}{8} & -\frac{127395662723}{1190957033472} & \frac{11484305}{2807808} & -\frac{31837466668875}{8631809448448} & \frac{23633379475167}{40731350834864} & & \\
\frac{11}{12} & -\frac{29667757954157}{65651506470144} & \frac{104671237}{9476352} & -\frac{4215138814594006525}{387256420118990016} & \frac{1366821681774917125}{1317839944877180109} & \frac{391082600}{2293800201} & \\
b & \frac{9789953}{89073390} & 0 & \frac{40381074286473}{79785138661750} & \frac{32899071073141444}{98334878315369685} & -\frac{58496}{20227515} & \frac{345492}{6606125} \\
b^* & \frac{97201106}{881826561} & 0 & \frac{318166544861}{630497193327} & \frac{52088110835257}{137317906103697} & -\frac{9294160}{109228581} & \frac{1}{11}
\end{bmatrix}$$

The coefficients are as follows:
c[2]=12/41,
c[3]=53/151,
c[4]=8003/9668,
c[5]=7/8,
c[6]=11/12,

a[2,1]=12/41,
a[3,1]=76903/547224,
a[3,2]=115169/547224,
a[4,1]=2011866719207/10844041507584,
a[4,2]=-21792911494331/10844041507584,
a[4,3]=2396462600599/903670125632,
a[5,1]=-127395662723/1190957033472,
a[5,2]=11484305/2807808,
a[5,3]=-31837466668875/8631809448448,
a[5,4]=23633379475167/40731350834864,
a[6,1]=-29667757954157/65651506470144,
a[6,2]=104671237/9476352,
a[6,3]=-4215138814594006525/387256420118990016,
a[6,4]=1366821681774917125/1317839944877180109,
a[6,5]=391082600/2293800201,

b[1]=9789953/89073390,
b[2]=0,
b[3]=40381074286473/79785138661750,
b[4]=32899071073141444/98334878315369685,
b[5]=-58496/20227515,
b[6]=345492/6606125,

b*[1]=97201106/881826561,
b*[2]=0,
b*[3]=318166544861/630497193327,
b*[4]=52088110835257/137317906103697,
b*[5]=-9294160/109228581,
b*[6]=1/11.

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