A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

The nodes of the scheme are:

$$c_2 = \frac{47}{228},\ c_3 = \frac{47}{152},\ c_4 = \frac{37}{46},\ c_5 = \frac{14}{15},\ c_6 = 1,\ c_7 = 1.$$

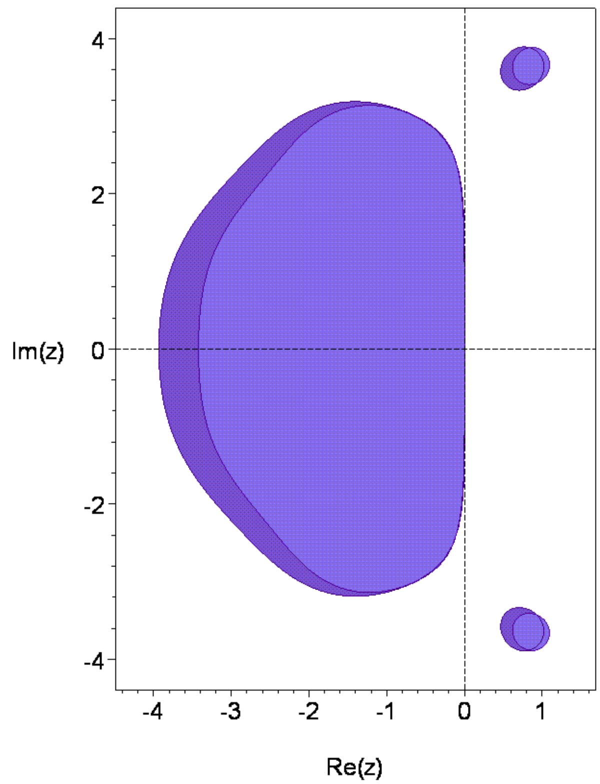
The principal error norm, that is, the 2-norm of the principal error terms is: $0.2592335271 \times 10^{(-3)}$.

The principal error norm of the order 4 embedded scheme is: $0.7685474335 \times 10^{(-3)}$.

The maximum magnitude of the linking coefficients is: 16.36725251.

The 2-norm of the linking coefficients is: 30.06070768.

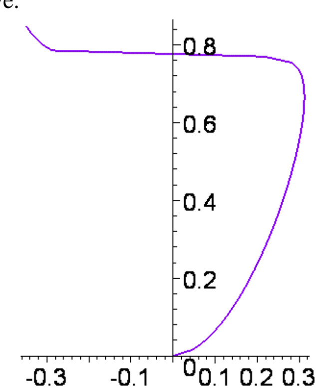
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.

The real stability intervals of the order 5 and 4 schemes are respectively $[-3.4217, 0]$ and $[-3.9338, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [0, 0.7704].

The Butcher tableau is as follows.

$$\left[\begin{array}{cccccccc}
\frac{47}{228} & \frac{47}{228} & & & & & & \\
\frac{47}{152} & \frac{47}{608} & \frac{141}{608} & & & & & \\
\frac{37}{46} & \frac{24466583}{26876903} & -\frac{185796573}{53753806} & \frac{90050082}{26876903} & & & & \\
\frac{14}{15} & \frac{3966704609}{960362750} & -\frac{382341883}{23360175} & \frac{8310310454516}{606546943875} & -\frac{10786398343}{20318910750} & & & \\
1 & \frac{625120811}{160033214} & -\frac{32896923}{2162611} & \frac{22409309668620}{1775657176381} & -\frac{785684025}{3720319438} & -\frac{12848625}{123987413} & & \\
1 & \frac{1152}{12173} & 0 & \frac{16747812352}{36467868465} & \frac{83392618}{153904941} & -\frac{475875}{1773058} & \frac{979}{5670} & \\
b & \frac{1152}{12173} & 0 & \frac{16747812352}{36467868465} & \frac{83392618}{153904941} & -\frac{475875}{1773058} & \frac{979}{5670} & \\
b* & \frac{13161933068}{140226569175} & 0 & \frac{64689707219693056}{140030233028618625} & \frac{309651895891498}{590968040075325} & -\frac{63325876995}{272329297394} & \frac{273499937}{1979255250} & \frac{4}{275}
\end{array}\right]$$

The coefficients are as follows:
c[2]=47/228,
c[3]=47/152,
c[4]=37/46,
c[5]=14/15,
c[6]=1,
c[7]=1,

a[2,1]=47/228,
a[3,1]=47/608,
a[3,2]=141/608,
a[4,1]=24466583/26876903,
a[4,2]=-185796573/53753806,
a[4,3]=90050082/26876903,
a[5,1]=3966704609/960362750,
a[5,2]=-382341883/23360175,
a[5,3]=8310310454516/606546943875,
a[5,4]=-10786398343/20318910750,
a[6,1]=625120811/160033214,
a[6,2]=-32896923/2162611,
a[6,3]=22409309668620/1775657176381,
a[6,4]=-785684025/3720319438,
a[6,5]=-12848625/123987413,
a[7,1]=1152/12173,
a[7,2]=0,
a[7,3]=16747812352/36467868465,
a[7,4]=83392618/153904941,
a[7,5]=-475875/1773058,
a[7,6]=979/5670,

b[1]=1152/12173,
b[2]=0,
b[3]=16747812352/36467868465,
b[4]=83392618/153904941,
b[5]=-475875/1773058,
b[6]=979/5670,

b*[1]=13161933068/140226569175,
b*[2]=0,
b*[3]=64689707219693056/140030233028618625,
b*[4]=309651895891498/590968040075325,
b*[5]=-63325876995/272329297394,
b*[6]=273499937/1979255250,
b*[7]=4/275.

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