A 6 stage, order 5 Runge-Kutta scheme with a 7 stage order 4 FSAL embedded scheme

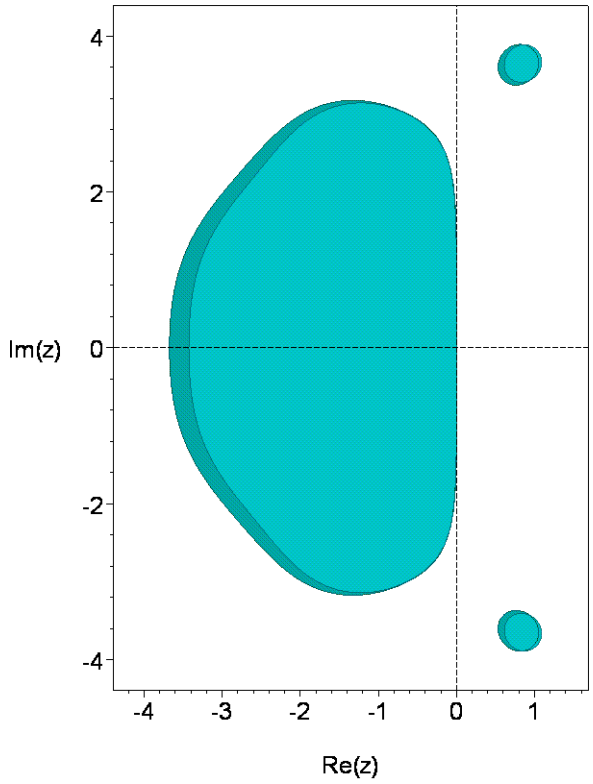
The nodes of the scheme are:

$$c_2 = \frac{7}{33},\ c_3 = \frac{7}{22},\ c_4 = \frac{8}{9},\ c_5 = \frac{58}{61},\ c_6 = 1,\ c_7 = 1.$$

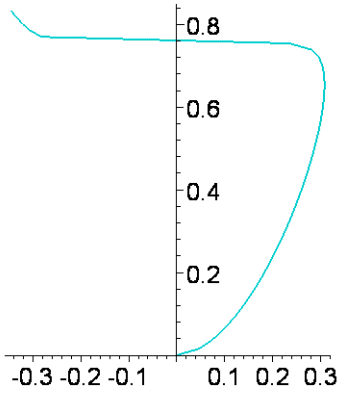
The principal error norm, that is, the 2-norm of the principal error terms is: $0.1898359780 \times 10^{(-3)}$.
Note: The scheme satisfies 9 of the principal error conditions.

The principal error norm of the order 4 embedded scheme is: $0.5626059950 \times 10^{(-3)}$.
The maximum magnitude of the linking coefficients is: 9.962944344.
The 2-norm of the linking coefficients is: 18.57288281.

The stability regions for the two schemes are shown in the following picture.



The stability region of the order 4 scheme appears in the darker shade.
The real stability intervals of the order 5 and 4 schemes are respectively [−3.4272, 0] and [−3.6757, 0].
The following picture shows the result of distorting the boundary curve of the stability region of the order 5 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval: [0, 0.7561].

The Butcher tableau is as follows.

$$\left[\begin{array}{cccccccc}
\frac{7}{33} & \frac{7}{33} & & & & & & \\
\frac{7}{22} & \frac{7}{88} & \frac{21}{88} & & & & & \\
\frac{8}{9} & \frac{44776}{35721} & -\frac{57376}{11907} & \frac{159104}{35721} & & & & \\
\frac{58}{61} & \frac{3756797837}{1824019316} & -\frac{3638341740}{456004829} & \frac{357735251280}{51528545677} & -\frac{305128053}{4206411892} & & & \\
1 & \frac{62143187}{23997848} & -\frac{1030557}{103439} & \frac{27950789460}{3307875781} & \frac{1366875}{32441848} & -\frac{139593315}{1178098436} & & \\
1 & \frac{6247}{64960} & 0 & \frac{72443668}{151100775} & \frac{938223}{614720} & -\frac{567679481}{301361040} & \frac{2111}{2700} & \\
b & \frac{6247}{64960} & 0 & \frac{72443668}{151100775} & \frac{938223}{614720} & -\frac{567679481}{301361040} & \frac{2111}{2700} & \\
b* & \frac{19538957}{205598400} & 0 & \frac{2853621408}{5904122875} & \frac{933255351}{648529600} & -\frac{182020376539}{105978632400} & \frac{7217509}{10444500} & \frac{1}{110}
\end{array}\right]$$

The coefficients are as follows:
c[2]=7/33,
c[3]=7/22,
c[4]=8/9,
c[5]=58/61,
c[6]=1,
c[7]=1,

a[2,1]=7/33,
a[3,1]=7/88,
a[3,2]=21/88,
a[4,1]=44776/35721,
a[4,2]=-57376/11907,
a[4,3]=159104/35721,
a[5,1]=3756797837/1824019316,
a[5,2]=-3638341740/456004829,
a[5,3]=357735251280/51528545677,
a[5,4]=-305128053/4206411892,
a[6,1]=62143187/23997848,
a[6,2]=-1030557/103439,
a[6,3]=27950789460/3307875781,
a[6,4]=1366875/32441848,
a[6,5]=-139593315/1178098436,
a[7,1]=6247/64960,
a[7,2]=0,
a[7,3]=72443668/151100775,
a[7,4]=938223/614720,
a[7,5]=-567679481/301361040,
a[7,6]=2111/2700,

b[1]=6247/64960,
b[2]=0,
b[3]=72443668/151100775,
b[4]=938223/614720,
b[5]=-567679481/301361040,
b[6]=2111/2700,

b*[1]=19538957/205598400,
b*[2]=0,
b*[3]=2853621408/5904122875,
b*[4]=933255351/648529600,
b*[5]=-182020376539/105978632400,
b*[6]=7217509/10444500,
b*[7]=1/110.

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