Lawson's 6 stage, order 5 Runge-Kutta scheme

See : An Order Five Runge Kutta Process with Extended Region of Stability, J. Douglas Lawson,
Siam Journal on Numerical Analysis, Vol. 3, No. 4, (Dec., 1966) pages 593-597.

The nodes of the scheme are:

$$c_2 = \frac{1}{12},\ c_3 = \frac{1}{4},\ c_4 = \frac{1}{2},\ c_5 = \frac{3}{4},\ \ c_6 = 1.$$

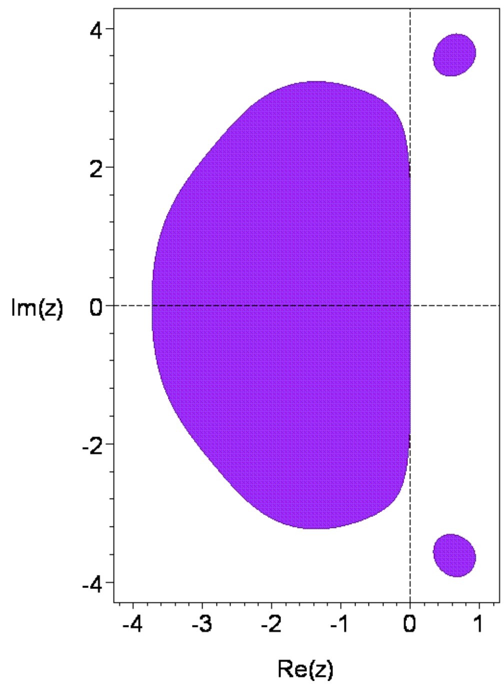
The principal error norm, that is, the 2-norm of the principal error terms is: $\frac{1}{7200} \simeq 0.1388888889 \times 10^{(-3)}$.

Note: 6 of the 20 principal error conditions are satisfied by the scheme.

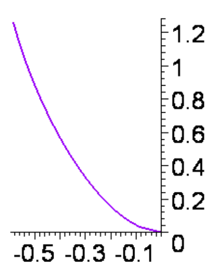
The maximum magnitude of the linking coefficients is: $\frac{66}{35} \simeq 1.885714286$.

The 2-norm of the linking coefficients is: $\frac{\sqrt{43113634}}{1680} \simeq 3.908391261$.

The stability region for the scheme is shown in the following picture.



The real stability interval of the scheme is $[-3.7344, 0]$.
The following picture shows the result of distorting the boundary curve of the stability region of the scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis only at the origin.

The Butcher tableau for the scheme is as follows.

$$\begin{bmatrix} \frac{1}{12} & \frac{1}{12} & & & & & \\ \frac{1}{4} & -\frac{1}{8} & \frac{3}{8} & & & & \\ \frac{1}{2} & \frac{3}{5} & -\frac{9}{10} & \frac{4}{5} & & & \\ \frac{3}{4} & \frac{39}{80} & -\frac{9}{20} & \frac{3}{20} & \frac{9}{16} & & \\ 1 & -\frac{59}{35} & \frac{66}{35} & \frac{48}{35} & -\frac{12}{7} & \frac{8}{7} & \\ & \frac{7}{90} & 0 & \frac{16}{45} & \frac{2}{15} & \frac{16}{45} & \frac{7}{90} \end{bmatrix}$$

The coefficients are as follows:
c[2]=1/12,
c[3]=1/4,
c[4]=1/2,
c[5]=3/4,
c[6]=1,

a[2,1]=1/12,
a[3,1]=-1/8,
a[3,2]=3/8,
a[4,1]=3/5,
a[4,2]=-9/10,
a[4,3]=4/5,
a[5,1]=39/80,
a[5,2]=-9/20,
a[5,3]=3/20,
a[5,4]=9/16,
a[6,1]=-59/35,
a[6,2]=66/35,
a[6,3]=48/35,
a[6,4]=-12/7,
a[6,5]=8/7,

b[1]=7/90,
b[2]=0,
b[3]=16/45,
b[4]=2/15,
b[5]=16/45,
b[6]=7/90.

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