

A 22 stage combined order 9 and 10 Runge-Kutta scheme

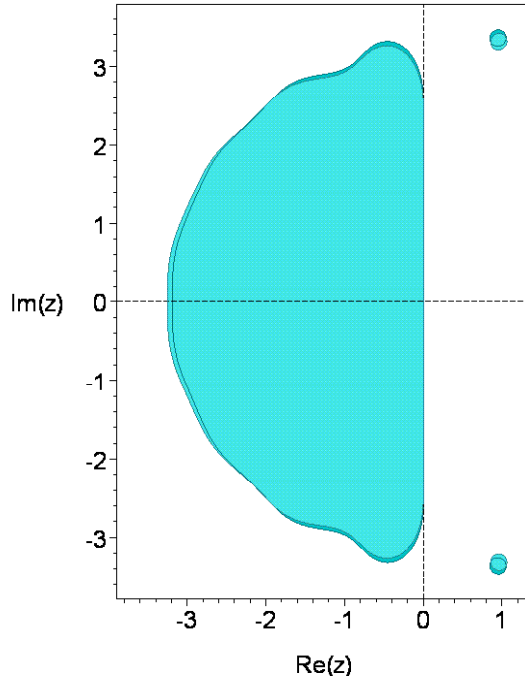
The principal error norm of the order 10 scheme, that is, the 2-norm of the principal error terms is: $0.7132554307 \times 10^{(-7)}$.
Note: 681 of the 1842 error terms are (effectively) zero.

The principal error norm of the embedded order 9 scheme is: $0.3543052193 \times 10^{(-6)}$.

The maximum magnitude of the linking coefficients is 6.592115791.

The 2-norm of the linking coefficients is 17.98192916.

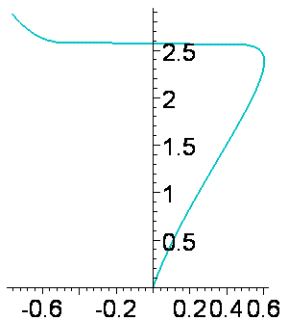
The stability regions for the two schemes are shown in the following picture.



The stability region of the order 9 scheme appears in the darker shade.

The real stability intervals of the order 10 and 9 schemes are respectively $[-3.2531, 0]$ and $[-3.1914, 0]$.

The following picture shows the result of distorting the boundary curve of the stability region of the order 10 scheme horizontally by taking the 11th root of the real part of points along the curve.



The stability region intersects the nonnegative imaginary axis in the interval $[0, 2.5697]$.

The coefficients correct to 85 digits are as follows.

c[2]=.2184542180095079835071455058295355977052891772686145691881287914864137982646025817860,
c[3]=.3276813270142619752607182587443033965579337659029218537821931872296206973969038726789,
c[4]=.4915219905213929628910773881164550948369006488543827806732897808444310460953558090184,
c[5]=.2073884728764196389262135702169880019340496964562935465039603023285685527378891693144,
c[6]=.5809550101170356824256657684472613818278876807365996395879202418770027297926200337654,
c[7]=.3378151260504201680672268907563025210084033613445378151260504201680672268907563025210,
c[8]=.4544536822241318177254571339172610141630759214200061515992212212409243663230446598427,
c[9]=.7455232760409578335045732927006396878799300126990640497421075028780763720414858728060,
c[10]=.1982944414756356991048607955831308536354571068278436481829163670965357890739794006642,
c[11]=.5175,
c[12]=.6702708700663741304377363531350821660615484781076307917000954798984486444618610938811,
c[13]=.77625,
c[14]=.3822843822843822843822843822843822843822843822843822843822843822843822843822843822844,
c[15]=.1105675146771037181996086105675146771037181996086105675146771037181996086105675146771,
c[16]=.8920187793427230046948356807511737089201877934272300469483568075117370892018779342723,
c[17]=.9771573604060913705583756345177664974619289340101522842639593908629441624365482233503,
c[18]=.4726426076833527357392316647264260768335273573923166472642607683352735739231664726426,
c[19]=.6000805477245267821184051550543697140555779299234796616995569875151026983487716472010,
c[20]=.9914529914529914529914529914529914529914529914529914529914529914529914529914530,
c[21]=1.,
c[22]=1.,

a[2,1]=.2184542180095079835071455058295355977052891772686145691881287914864137982646025817860,
a[3,1]=.8192033175356549381517956468607584913948344147573046344554829680740517434922596816973e-1,
a[3,2]=.2457609952606964814455386940582275474184503244271913903366448904222155230476779045092,
a[4,1]=.1228804976303482407227693470291137737092251622135956951683224452111077615238389522546,
a[4,2]=0.,
a[4,3]=.3686414928910447221683080410873413211276754866407870855049673356333232845715168567638,
a[5,1]=.1164691491511012239537993895753631423015075199584078911869286353023391531058784508211,
a[5,2]=0.,
a[5,3]=.1415024642665197815069814909917318008838862351479722090285340883254369387305191074218,
a[5,4]=-.5058314054120136653456731035010694125134405865008655371150242129920753909850838892848e-1,
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a[6,4]=.2173985478806202942240889146337235614522941954898205252907826202244616090358936415467,
a[6,5]=.2984649727190193579283373561946200795617681853430338986974167175996034563431675255250,
a[7,1]=.7112167058948449559972437745291251302301998618575747929447252805205340506969237711631e-1,
a[7,2]=0.,
a[7,3]=0.,
a[7,4]=0.,
a[7,5]=.2620078611542798878142155065734472981369668753099696943690703779802660032596688446619,
a[7,6]=.4685594306655784653287006729942709848416499848810641462507514135747818561395080742824e-2,
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a[8,3]=0.,
a[8,4]=0.,
a[8,5]=.2162499004631919931721335492246112301817759406189855036193836300395852486927949073527,
a[8,6]=.1593590545416229688047003951136306276308382833842549675637500286264541868034051949201e-1,
a[8,7]=.1455183327259091186070887761992170007743009529261485945974139005404389073800559991405,
a[9,1]=.9373408329646649104587686775176964978370974684271239105793932145754493274293133794592e-1,
a[9,2]=0.,
a[9,3]=0.,
a[9,4]=0.,
a[9,5]=0.,
a[9,6]=.5856103617170312801793383881265389368733956755406950888289958681340881566203997185682,
a[9,7]=.7920709286272682345169655495666789082385343327455120754698291270282904071120088715217,

a[9,8]=-.7258920975998081722376075127443478070157097424298555056146568137418471244338540552298,
a[10,1]=.8923572736943979359212465026656706617224264656322763272720964198146031192443830337050e-1,
a[10,2]=0.,
a[10,3]=0.,
a[10,4]=0.,
a[10,5]=0.,
a[10,6]=.2887654872742642283550523996942162287400901726033714256327335427920126730332619741850,
a[10,7]=.4619614431515204150919059437319450363329051767621119917584303277439657433539182334270,
a[10,8]=-.5985565565536507075622101429954146160499414045842243323682092085472093586759817971410,
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a[11,3]=0.,
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a[11,5]=0.,
a[11,6]=0.,
a[11,7]=0.,
a[11,8]=.1734081111923846212346610109997869706575573690860928251398657589459534207225697200913,
a[11,9]=-.1329848597546858329968102242796422163661917560290326458187513105129932344987186823619e-2,
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a[12,5]=0.,
a[12,6]=0.,
a[12,7]=0.,
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a[13,3]=0.,
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a[13,5]=0.,
a[13,6]=0.,
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a[14,2]=0.,
a[14,3]=0.,
a[14,4]=0.,
a[14,5]=0.,
a[14,6]=0.,
a[14,7]=0.,
a[14,8]=0.,
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a[18,16]=.1226062391970877914749997306216330664433795471715860812488387526407184146129677216888,
a[18,17]=-.2842815761481977469580701607513000599666395580254806752482327428602114050821070719599e-1,
a[19,1]=.1010970852075341203880190191006102645724855276568229748629566967650053981829664420677,
a[19,2]=0.,
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a[19,4]=0.,
a[19,5]=0.,
a[19,6]=0.,
a[19,7]=0.,
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a[19,10]=.5613754225178724054814530524992960771099371015556758438475643385140637692818755727903,
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a[19,14]=-.8940804813395937430063744506942506700155221194106897425933891507362686665515723406737e-1,
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a[19,17]=-.2641159908391797708179717732855551253685670920778528161010219548610383931527649127101e-1,
a[19,18]=-.8566368056442731028193168221601726460821205318327373118613572117292207584833230643611e-1,
a[20,1]=-.3756786453598955156381295023657603018017983596261589443254234724267442502509051411985,
a[20,2]=0.,
a[20,3]=0.,
a[20,4]=0.,
a[20,5]=0.,
a[20,6]=0.,
a[20,7]=0.,
a[20,8]=0.,
a[20,9]=-2.450122089272237415542143457491503614988906924204946830128637714991221143092866149291,
a[20,10]=-1.291417779301475040315145604068899722621769620341378063556352840209283333902923697064,
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a[20,12]=-.1875170157439401784690730466779540914395127998272848073073325439944722110642929462542,
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a[20,19]=-3.346060975395236616311839674536896490185601106709970593714830003347702027368235184914,
a[21,1]=-.3489096904942705525941965308532521623960037434723186184362947297534792605982710353117,
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a[21,4]=0.,
a[21,5]=0.,
a[21,6]=0.,
a[21,7]=0.,
a[21,8]=0.,
a[21,9]=-3.111089745437775323074369816338589211169218129249007828527465845312446145797861590440,
a[21,10]=-.6951415273708152879147075266970882714562984072493231826489802870759087247204125153980,
a[21,11]=5.341622083430723179684928184313469958406467535674389555427138733956674403962343101434,
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a[21,13]=2.137823030702064694260595546500666529806827669540682980469810839545545293452132886010,
a[21,14]=-3.146752166302723093681043025274376033817557611519092096831424307607146503892301450420,
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a[21,16]=.8127931897459860971317231950675872129264366060894158904308025679686941225239570336145,
a[21,17]=-.2908072531017927384672100763769591816645324611971887595127700441014568784368742585001,
a[21,18]=2.616720125366438534132512271186617679162725298787743272613211376702519647613882120059,
a[21,19]=-6.592115791300909741311510815578290956921808813583194857283823364175932015226748117335,

a[21,20]=- .1763724520558459093495126494388724460310502301268061108329758239610996064521939830725e-1,
a[22,1]=- .3501518246653591094298524897559170848887272237237690751344243169151287635133982738027,
a[22,2]=0.,
a[22,3]=0.,
a[22,4]=0.,
a[22,5]=0.,
a[22,6]=0.,
a[22,7]=0.,
a[22,8]=0.,
a[22,9]=-3.102572667985050019435714963026276866679448642033047018252551100604565380743157412153,
a[22,10]=- .7089070045036944395352732193464063886491311874005060293599170898708585766100525112663,
a[22,11]=5.329777431614684588882588506365780121821875751170238779725707622449341805417256633900,
a[22,12]=2.716180945292403663284578725805530227147786577798169039711863864775518300343983312389,
a[22,13]=2.161517223925592281805033254338965145748026497076128918755018096412145584461892115898,
a[22,14]=-3.120266861350477157724474048868356921346385902909040890897599111279336798448052738888,
a[22,15]=1.519241430999458193316587921277653155425346252266826468205110973034625916978393049859,
a[22,16]=.8020380714443328721618368063864093995387748912202124970587855828184710427510527088975,
a[22,17]=- .2885001942210796458575781164106895686965144268346964549367020960276059430170015596842,
a[22,18]=2.585771817179182248058617150521610614963671584309568975729745410501649368723359052278,
a[22,19]=-6.526712053851098940276076677844200530461852471057430540136374001956477788072878292646,
a[22,20]=- .174163138788945352502728494441013039234216998826546704686638333377876827139608478230e-1,
a[22,21]=0.,

b[1]=-3082802728716466354866131900356163881429369782871485293954942328832169782844426032620e-1,
b[2]=0.,
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b[4]=0.,
b[5]=0.,
b[6]=0.,
b[7]=0.,
b[8]=0.,
b[9]=0.,
b[10]=0.,
b[11]=0.,
b[12]=-1.312016170791286152673526744872266847284875686903956215174197432309311643485301831369,
b[13]=-8392136168254867926404344877321794737572715999217552860667951068118531791815589899256,
b[14]=-5850852294034988787550646883200680772588557359409020872655169137056901956573473254514,
b[15]=.1819369811405084288206498145088261292259147702857263231499342098211808141410293876619,
b[16]=-4257933659006158544582003914778792998823878664438058493888780996336877415311167415662,
b[17]=2.319732334075545696600501873633794462667088193935546108462899618247877013446957810865,
b[18]=-6940421332627756542006690971300476069005132380264175651054445505427461960861421581986,
b[19]=1.456876093065142187479382506381596747918566471805351039490511394532129237471913308075,
b[20]=-3.845208589984745489244538893258302720929404935174417160602861447309284698701193083411,
b[21]=1.863387978142076502732240437158469945355191256830601092896174863387978142076502732240,
b[22]=0.,

b*[1]=.3072798718815191487252640802669776134320204564009876204582409544565381146923057660751e-1,
b*[2]=0.,
b*[3]=0.,
b*[4]=0.,
b*[5]=0.,
b*[6]=0.,
b*[7]=0.,
b*[8]=0.,
b*[9]=0.,
b*[10]=0.,
b*[11]=0.,
b*[12]=-1.152580767662478030705311715973597147360033208332273829897357839055966320654258340159,

b*[13]=.7416493298999577539323576892675799100052325957600899545882900243724841318076356101703,
b*[14]=.5723967468885128709768747046235710937418199615700457777421659817834200321468049593942,
b*[15]=.1823933606905595379811314770104132466132817030898646419583954799261545521759267244650,
b*[16]=-.3626410780927759822398498783511672491471638817247374382635865415914492127900383201158,
b*[17]=2.203121491080439977912798209428968181244958159637525190837474401126330904571874121742,
b*[18]=-.6520111834051860439615188353079149009338039356345062217215109661870225083317315235351,
b*[19]=1.330307398998584754306781976991671317478388036698139307669981224778500790768548465660,
b*[20]=-3.717924689094538682900351439224994142810442880213018074784237264106878110988553677738,
b*[21]=0.,
b*[22]=1.824561403508771929824561403508771929824561403508771929824561403508771929824561403509.

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